Deductive Reasoning
Propositional Logic

Chapter Objectives

Connectives and Truth Values

You will be able to

- understand the purpose and uses of propositional logic.
- understand the meaning, symbols, and uses of the four logical connectives—conjunction, disjunction, negation, and conditional.
- define *statement* and explain the distinction between simple and compound statements.
- translate simple statements into symbolic form.
- construct a truth table and use it to test the validity of arguments.
- identify the situations in which conjunctions, disjunctions, negations, and conditionals are true or false.
- understand the structure of conditional statements and the various ways in which they can be expressed.

Checking for Validity

You will be able to

- determine the validity of very simple arguments using truth tables.
- use parentheses effectively in expressing statements in symbolic form.
- use the short method to evaluate complex arguments.

Chapter Summary

In **propositional logic** we use symbols to stand for the relationships between statements—that is, to indicate the form of an argument. These relationships are made possible by **logical connectives** such as **conjunction** (and), **disjunction** (or), **negation** (not), and **conditional** (If . . . then . . .). Connectives are used in **compound statements**, each of which is composed of at least two **simple statements**. A statement is a sentence that can be either true or false.

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To indicate the possible truth values of statements and arguments, we can construct **truth tables**, a graphic way of displaying all the truth value possibilities. A conjunction is false if at least one of its statement components (**conjuncts**) is false. A disjunction is still true even if one of its component statements (**disjuncts**) is false. A negation is the denial of a statement. The negation of any statement changes the statement's truth value to its contradiction (false to true and true to false). A conditional statement is false in only one situation—when the antecedent is true and the consequent is false.

The use of truth tables to determine the validity of an argument is based on the fact that it's impossible for a valid argument to have true premises and a false conclusion. A basic truth table consists of two or more **guide columns** listing all the truth value possibilities, followed by a column for each premise and the conclusion. We can add other columns to help us determine the truth values of components of the argument.

Some arguments are complex when variables and connectives are combined into larger compounds and when the number of variables increases. To prevent confusion, we can use parentheses in symbolized arguments to show how statement or premise components go together. You can check the validity of arguments not only with truth tables but also with the short method. In this procedure, we try to discover if there is a way to make the conclusion false and the premises true by assigning various truth values to the argument's components.

Answers to Select Textbook Exercises

Please note: These answers are for some of the questions that were not answered in Appendix B of *The Power of Critical Thinking*, Fifth Canadian Edition.

Exercise 7.1

2. * Conditional. Components: I can get a discount on plane tickets to Japan, I'll have to vacation in Paris; →

8. * Conjunction. Components: The chief executive officer of the company recently resigned, there had been rumours of "financial irregularities" at the company.

Exercise 7.2

3. * $\sim y \rightarrow x$ (Note that "unless" is translated as "if not.")

10. * $\sim y$ & $\sim z$

16. $p \vee q$

Exercise 7.3

1. **True** (A disjunction is true even if one of the disjuncts is false.)

9. **True** (Conditionals are true whenever their antecedents are false; 'it is not the case that dogs are not reptiles' is false.)

Exercise 7.4

1. **False** (Conditional statements are false when their antecedent is true and their consequent is false)

4. **False** (A disjunction is false if *both* disjuncts are false—here, both $\sim b$ and $\sim c$ are false, since b and c are assumed to be true in this assignment.)

7. **True** (Both of the conjuncts are true)

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Exercise 7.5

1. * If alligators are reptiles, then alligators have scales.
6. * Alice is here, and Quentin is not here.
9. * If Bob drove home, then Cathy will not leave.

Exercise 7.6

1. * $p \rightarrow q$
8. * $\sim p \mathbin{\&} \sim q$

Exercise 7.7

1. It is currently winter in Australia ∨ It is currently winter in Canada.

   $p \vee q$

p	***q***	***p* ∨ *q***
T	T	T
T	F	T
F	T	T
F	F	F

9. ~~Dogs are mammals ∨ Snakes are reptiles

   $\sim\sim p \vee q$

p	***q***	***~~p***	***~~p* ∨ q**
T	T	T	T
T	F	T	T
F	T	F	T
F	F	F	F

Exercise 7.8

4. **Valid**

   a & *b*
   ~*a*
   ∴ *b*

a	***b***	***a* & *b***	**~*a***	***b***
T	T	T	F	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	F

10. **Valid**

   $p \rightarrow q$
   $\therefore p \rightarrow (p \mathbin{\&} q)$

p	***q***	***p* & *q***	***p* → *q***	***p* → (*p* & *q*)**
T	T	T	T	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

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Exercise 7.9

1. * Invalid

   (For the analysis below, to save space we've left out the premise about the conditions under which Heather will go, since it turns out to be irrelevant. There are no lines in which the two premises about Joanne's preferences are both True, which is sufficient to guarantee that the argument is invalid.)

   $j \rightarrow (e \,\&\, {\sim}h)$

   $j \rightarrow ({\sim}e \,\&\, {\sim}h)$

   $\therefore j \,\&\, ({\sim}e \,\&\, {\sim}h)$

j	e	h	$e \,\&{\sim} h$	$j \rightarrow (e \,\&{\sim} h)$	$({\sim}e \,\&\, {\sim}h)$	$j \rightarrow ({\sim}e \,\&\, {\sim}h)$	$j \,\&\, ({\sim}e \,\&\, {\sim}h)$
T	T	T	F	F	T	T	F
T	T	F	T	T	F	F	F
T	F	T	F	F	F	F	F
T	F	F	F	F	F	F	T
F	F	T	F	T	F	T	F
F	T	F	T	T	F	T	F
F	F	T	F	T	F	T	F
F	F	F	F	T	F	T	F

Study Questions

1. What is propositional logic?
2. What are logical connectives?
3. What are the symbols for, and the meaning of, the four logical connectives?
4. What is a simple statement? A compound statement?
5. What is a truth table?
6. In what situation is a conjunction false (or true)?
7. What is a disjunct?
8. Under what circumstances is a disjunction true (or false)?
9. What is the logical symbol for negation?
10. Under what circumstance(s) is a conditional false?
11. What is the short method of argument evaluation?

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12. "It is not the case that either Alice walks home or Jan walks home." What is the symbolization for this statement?

13. How is a hypothetical syllogism expressed in symbolic form?

14. How is *modus ponens* expressed in symbolic form?

15. How is a disjunctive syllogism expressed in symbolic form?

16. Why do "*p* if not *q*," and "*p* unless *q*" have the same symbolization? What is the correct symbolization?

17. Why do "if *p* then *q*," and "*p* only if *q*" have the same symbolization? What is the correct symbolization?

18. Why does "if *p* then *q*" have the same symbolization as "Either not *p*, or *q*"? What is the correct symbolization?

19. Explain the difference between *modus ponens* and denying the antecedent. Why is the former valid while the latter is invalid? Use truth-tables to illustrate.

20. Explain the difference between *modus tollens* and affirming the consequent. Why is the former valid while the latter is invalid? Use truth-tables to illustrate.

Self-Assessment Quiz

Scroll down for answers.

Translation

Translate each sentence into standard form, using the assignment of variables indicated.

1. You can have dessert only if you eat all your peas. (*P* = You can have dessert; *Q* = You eat all your peas.)

2. Either Bob and Abdul or Meng and Janice proceed to the final game. (*P* = Bob proceeds to the final game; *Q* = Abdul proceeds to the final game; *R* = Meng proceeds to the final game; *S* = Janice proceeds to the final game.)

3. If either Raven or Juanita get hired, then we will have gender equality in our department. (*P* = Raven gets hired; *Q* = Juanita gets hired; *R* = We will have gender equality in our department.)

4. Ali will either win or he won't. (*P* = Ali will win.)

5. Sebastian Giovinco or Jonathan Osorio scored, but not both. (*P* = Sebastian Giovinco scored; *Q* = Jonathan Osorio scored)

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6. Neither McDavid nor Draisatl scored. (*P* = McDavid Scored; *Q* = Draisatl scored)

7. It is not the case that both the Raptors and the Pistons will win. (*P* = The Raptors will win; *Q* = the Pistons will win.)

8. If Alan Turing did not invent the computer, then Charles Babbage did. (*P* = Alan Turing invented the computer; *Q* = Charles Babbage invented the computer)

9. It's not the case that if the Patriots make the playoffs, then they will win the Super Bowl. (*P* = The Patriots make the playoffs; *Q* = The Patriots win the Super Bowl.)

10. If either Raonic or Nadal win the US Open, then Federer will not. (*P* = Raonic will win the US Open; *Q* = Nadal will win the US Open; *R* = Federer will win the US Open)

11. If neither Daisy Johnson nor Richard Powers win the Man Booker Prize, then Anna Burns will. (*P* = Daisy Johnson will win the Man Booker Prize; *Q* = Richard Powers will win the Man Booker Prize; *R* = Anna Burns will win the Man Booker Prize)

12. Phil Mickelson will win the Masters, only if neither Tom Watson nor Tiger Woods do. (*P* = the Phil Mickelson will win the Masters; *Q* = Tom Watson will win the Masters; *R* = Tiger Woods will win the Masters.

13. Tavares will score the most goals only if neither Laine nor Ovechkin do. (*P* = Tavares will score the most goals; *Q* = Laine will score the most goals; *R* = Ovechkin will score the most goals)

14. Meryl Streep will win the Oscar if Frances McDormand doesn't. (*P* = Meryl Streep will win the Oscar; *Q* = Frances McDormand will win the Oscar)

15. Denis Mukwege will win the Nobel Peace Prize unless Nadia Murad does (*P* = Denis Mukwege will win the Nobel Peace Prize; *Q* = Nadia Murad will win the Nobel Peace Prize)

Checking for Validity

Construct a truth table for each of the following arguments and show whether the argument is valid or invalid.

1.

$P \lor Q$
$\underline{P}$
$\sim Q$

2.

P
$\underline{\sim Q}$
$P \lor \sim Q$

3.

$R \rightarrow Q$
$\underline{Q \rightarrow P}$
$\sim P \rightarrow \sim R$

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4. $R \lor Q$
$\sim R \rightarrow P$
$\underline{\sim Q \rightarrow P}$
$\sim P$

5. $(P \,\&\, Q) \rightarrow R$
$\underline{\sim R}$
$\sim P \lor \sim R$

6. $\sim(P \,\&\, Q) \rightarrow (Q \lor R)$
$\underline{\sim R}$
$\sim P$

7. $\sim R$
$\sim P$
$\underline{P \lor Q}$
$\sim R$

8. $(P \,\&\, Q) \rightarrow (R \lor \sim Q)$
$\underline{\sim R}$
$P \,\&\, Q$

9. $(P \lor R)$
$P \rightarrow \sim Q$
$\underline{R \rightarrow \sim Q}$
$\sim Q$

10. $(P \rightarrow Q) \rightarrow (P \rightarrow R)$
$\sim(P \rightarrow Q)$
$\underline{\sim R}$
P

11. $P \rightarrow Q$
$\underline{\sim P}$
$\sim Q$

12. $P \rightarrow Q$
$\underline{Q}$
P

13. $P \rightarrow \sim Q$
$\underline{Q}$
P

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Checking for Validity (Short Method)

Translate each of the following arguments into symbolic form and use the short method to determine validity.

1. Either Gary or his wife will attend the meeting. Gary cannot attend. Therefore, his wife will attend.

2. If either the Canucks win their series against the Kings, or the Jets win their series against the Black Hawks, then the Canadiens will win the cup. But there is no way that the Canadiens will win the cup. So, both the Jets and the Canucks will lose their series.

3. Either Archana committed the crime, or it was Bruce again. If Bruce committed the crime, then Penny must be innocent. However, we know that Penny is guilty. Therefore, Archana is guilty too.

4. If the explosion were an act of terrorism, then someone would have taken responsibility. The YST have taken responsibility for the explosion. So, the explosion was indeed an act of terrorism.

5. The planet is getting warmer. If it is getting warmer, then this is either the result of human activity or the result of non-human factors. It is surely a matter of human activity. Therefore, global warming is not caused by natural factors.

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Answers to Self-Assessment Quiz

Translation

1. $P \rightarrow Q$
2. $(P \,\&\, Q) \lor (R \,\&\, S)$
3. $(P \lor Q) \rightarrow R$
4. $P \lor \sim P$
5. (P V Q) & ~(P & Q)
6. ~P & ~ Q; alternate, equivalent translation: ~(P V Q)
7. ~(P & Q); alternate, equivalent translation: ~P V ~Q
8. ~P → Q
9. ~(P → Q)
10. (P V Q) → ~R
11. ~(P V Q) → R; alternate equivalent translation: (~P & ~Q) → R
12. P → ~(Q V R); alternative, equivalent translation: P → (~Q & ~R)
13. P → ~(Q V R); alternative, equivalent translation: P → (~Q & ~R)
14. ~Q → P
15. ~Q → P

Checking for Validity

1. **Invalid** (Check first row)

P	***Q***	***P* ∨ *Q*** **Premise**	***P*** **Premise**	**~*Q*** **Conclusion**
T	T	T	T	F
T	F	T	T	T
F	T	T	F	F
F	F	F	F	T

2. **Valid** (No row has true premises and a false conclusion.)

P	***Q***	***P*** **Premise**	**~*Q*** **Premise**	***P* ∨ ~*Q*** ***Conclusion***
T	T	T	F	T
T	F	T	T	T
F	T	F	F	F
F	F	F	T	T

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3. **Valid**

P	*Q*	*R*	*R* → *Q* Premise	*Q* → *P* Premise	~*P* → ~*R* Conclusion
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	T
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	T	F	T
F	F	T	F	T	F
F	F	F	T	T	T

4. **Invalid**

P	*Q*	*R*	*R* ∨ *Q* Premise	~*R* → *P* Premise	~*Q* → *P* Premise	~*P* Conclusion
T	T	T	T	T	T	F
T	T	F	T	T	T	F
T	F	T	T	T	T	F
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	F	T
F	F	F	F	F	F	T

5. **Valid**

P	*Q*	*R*	(*P* & *Q*)	(*P* & *Q*) → *R* Premise	~*R* Premise	~*P* ∨ ~*R* Conclusion
T	T	T	T	T	F	F
T	T	F	T	F	T	T
T	F	T	F	T	F	F
T	F	F	F	T	T	T
F	T	T	F	T	F	T
F	T	F	F	T	T	T
F	F	T	F	T	F	T
F	F	F	F	T	T	T

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6. **Invalid**

P	***Q***	***R***	**~(*P* & *Q*)**	**(*Q* ∨ *R*)**	**~ (*P* & *Q*) → (*Q* ∨ *R*)** ***Premise***	**~*R*** ***Premise***	**~*P*** ***Conclusion***
T	T	T	F	T	T	F	F
T	T	F	F	T	T	T	F
T	F	T	T	T	T	F	F
T	F	F	T	F	F	T	F
F	T	T	T	T	T	F	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	F	T
F	F	F	T	F	F	T	T

7. **Valid** (Note that the truth table method is unnecessary—the first premise is identical to the conclusion!)

P	***Q***	***R***	**~*R*** **Premise**	**~*P*** **Premise**	***P* ∨ *Q*** **Premise**	**~*R*** **Conclusion**
T	T	T	F	F	T	F
T	T	F	T	F	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
F	T	T	F	T	T	F
F	T	F	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	F	T

8. **Invalid**

P	***Q***	***R***	**(*P* & *Q*)**	**(*R* ∨ ~*Q*)**	**(*P* & *Q*) → (*R* ∨ ~*Q*)** **Premise**	**~*R*** **Premise**	***P* & *Q*** **Conclusion**
T	T	T	T	T	T	F	T
T	T	F	T	F	F	T	T
T	F	T	F	T	T	F	F
T	F	F	F	T	T	T	F
F	T	T	F	T	T	F	F
F	T	F	F	F	T	T	F
F	F	T	F	T	T	F	F
F	F	F	F	T	T	T	F

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9. **Valid**

P	***Q***	***R***	***P* ∨ *R*** Premise	***P* → ~*Q*** Premise	***R* → ~*Q*** Premise	**~*Q*** Conclusion
T	T	T	T	F	F	F
T	T	F	T	F	T	F
T	F	T	T	T	T	T
T	F	F	T	T	T	T
F	T	T	T	T	F	F
F	T	F	F	T	T	F
F	F	T	T	T	T	T
F	F	F	F	T	T	T

10. **Valid**

P	***Q***	***R***	**(*P* → *Q*)**	**(*P* → *R*)**	**(*P* → *Q*) → (*P* → *R*)** *Premise*	**~(*P* → *Q*)** *Premise*	**~*R*** **(*P*)**	***P*** **(*C*)**
T	T	T	T	T	T	F	F	T
T	T	F	T	F	F	F	T	T
T	F	T	F	T	T	T	F	T
T	F	F	F	F	T	T	T	T
F	T	T	T	T	T	F	F	F
F	T	F	T	T	T	F	T	F
F	F	T	T	T	T	F	F	F
F	F	F	T	T	T	F	T	F

11. **Invalid** (On the third row, both of the premises are true but the conclusion is false; denying the antecedent)

P	***Q***	***P* → *Q*** Premise	**~ *P*** Premise	**~*Q*** Conclusion
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

12. **Invalid** (On the third row, both of the premises are true but the conclusion is false; affirming the consequent)

P	***Q***	***P* → *Q*** Premise	***Q*** Premise	***P*** Conclusion
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

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13. **Valid**

P	***Q***	***~Q***	***P → ~Q*** **Premise**	***Q*** **Premise**	***P*** **Conclusion**
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	T	T	F
F	F	T	T	F	F

Checking for Validity (Short Method)

1. *P* = Gary will attend the meeting.
   Q = Gary's wife will attend the meeting.

   P ∨ *Q*
   <u>~*P*</u>
   Q

   T F F
   P ∨ ***Q*** **~*P*** ***Q***
   T F F F

   Valid: There is no way to make the conclusion F while the premises are all T.

2. *P* = The Canucks win their series against the Kings.
   Q = The Jets win their series against the Black Hawks.
   R = The Canadians will win the cup.

   (*P* ∨ *Q*) → R
   <u>~R</u>
   ~*P* & ~*Q*

   T F F
   (*P* ∨ *Q*) → *R* **~*R*** **~*P* & ~*Q***
   T T T T T T

   Valid: The conclusion is F only if the second premise is also F.

3. *P* = Archana committed the crime.
   Q = Bruce committed the crime.
   R = Penny committed the crime.

   P ∨ *Q*
   Q → ~R
   <u>R</u>
   P

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T	T	F	F
P* ∨ *Q	***Q* → ~*R***	***R***	***P***
F T	T F		F

Valid

4. *P* = The explosion was an act of terrorism.
   Q = Someone has taken responsibility for the explosion.

   P → *Q*
   Q
   P

T	T	F
P* → *Q	***Q***	***P***
F T	T	F

Invalid: We can make all premises T but still have a F conclusion.

5. *P* = The planet is getting warmer.
   Q = The cause is human activity.
   R = The cause is natural factors.

   P
   P → (*Q* ∨ R)
   Q
   ~R

T	T	T	F
P	***P* → (*Q* ∨ *R*)**	***Q***	**~*R***
T	T T T	T	T

Invalid: All the premises can be T but the conclusion can be F.

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