Baronett, *Logic* (4th ed.)
Chapter Guide

Chapter 9: Predicate Logic

Predicate logic integrates the most powerful features of both categorical and propositional logic, thereby allowing for a more extended scope of argument analysis than either of the two can achieve individually.

A. Translating Ordinary Language

There are two types of statement in predicate logic: singular and quantified.

A **singular statement** is about a specific person, place, time, or object.

A **quantified statement** is about classes of things. Such statements are either universal or particular.

There are two elements in a singular statement: predicate and individual constant.

The **predicate** of a singular statement is the fundamental unit, and is translated with a capital letter, *A* to *Z*.

The subject of a singular statement is called an **individual constant** and is translated with a lowercase letter, *a* to *w*:

Ordinary Language	Predicate Logic Translation
Joe is a dog.	*Dj*

Individual variables, the lowercase letters, *x*, *y*, and *z*, are enlisted as placeholders in quantified statements. That's because quantified statements do not specify things, only classes of things. Things are included in, or excluded from, classes:

All dogs are mammals.
No dogs are cats.
Some dogs are beagles.
Some cats are not friendly animals.

Notice that quantifiers and classes are features of predicate logic borrowed from categorical logic. What is borrowed from propositional logic are the logical operators, ~, •, v, ⊃, ≡:

Ordinary Language	Statement Function
All dogs are mammals.	$Dx \supset Mx$

No dogs are cats.	$Dx \supset {\sim}Cx$
Some dogs are beagles.	$Dx \bullet Bx$
Some cats are not friendly animals.	$Cx \bullet {\sim}Fx$

The **statement functions** are expressions that do not make any universal or particular assertion about anything; therefore, they have no truth value. Their variables are *free*, which means we don't know how many things we're talking about.

"All" and "no" are universal quantifiers. They are translated as follows: (x). "Some" is a particular quantifier, and is translated as follows: $(\exists x)$.

Ordinary Language	Predicate Logic Translation
All dogs are mammals.	$(x)(Dx \supset Mx)$
No dogs are cats.	$(x)(Dx \supset {\sim}Cx)$
Some dogs are beagles.	$(\exists x)(Dx \bullet Bx)$
Some cats are not friendly animals.	$(\exists x)(Cx \bullet {\sim}Fx)$

Notice that the appearance of the quantifiers includes parentheses around what are otherwise statement functions. These parentheses tell us the **domain of discourse**, which is the set of individuals over which a quantifier ranges. The variables in the statement function are *bound* by the quantifier:

"For any *x*, if *x* is a dog, then *x* is a mammal."
"For any *x*, if *x* is a dog, then *x* is not a cat."
"There is at least one *x* that is a dog and a beagle."
"There is at least one *x* that is a cat and not a friendly animal."

B. Four New Rules of Inference

There are four quantifier rules of inference that allow you to remove or introduce a quantifier:

Universal Instantiation (UI): Consider what a universally quantified statement asserts, namely that the entirety of the subject class is contained within the predicate class. Therefore, any *instance* of a member in the subject class is also a member of the predicate class.

All dogs are mammals.
<u>Joe is a dog.</u>
Therefore, Joe is a mammal.

$(x)(Dx \supset Mx)$
$\underline{Dj\ \ }$
Mj

The only way MP can be employed is if we remove the universal quantifier, which, as we saw from the previous explanation, can be done by naming a member of the subject class in the universally quantified statement:

$(x)(Dx \supset Mx)$
$Dj \supset Mj$

In this case, we use the individual constant, *j*, because the statements need to match up if we are to use MP. Notice also that the instantiation of the individual constant, *j*, applies to the entire line.

Universal Generalization (UG): Moving from a universally quantified statement to a singular statement is not controversial. On the other hand, we can recognize pretty quickly that we can't go the other direction quite as easily. Consider the following statement: "Joe the dog is an American Staffordshire Terrier." We cannot infer from this statement that *all* dogs are American Staffordshire Terriers. So, when we want to make an inference to a universal statement, we may not do so from an individual constant:

Not Permitted:
$\underline{Sa}$
$(x)Sx$

Instead, the generalization must be made from a statement function, where the variable, by definition, could be *any entity in the relevant class of things*:

$Dx \supset Mx$
$(x)(Dx \supset Mx)$

If it's the case that entities *x* are members of the *D* class, then they're also members of the *M* class. This phrase, "entities *x*," suggests the quantity is not limited. Notice also that the generalization of the variable, *x*, applies to the entire line.

Existential Generalization (EG): Like **UI**, **EG** is a fairly straightforward inference. Take the sentence "Joe is an American Staffordshire Terrier dog." The sentence specifies an existing American Staffordshire Terrier. So, if Joe is one, it follows that at least one American Staffordshire Terrier exists:

$\underline{Aj}$
$(\exists x)Ax$

Notice also that the generalization to the variable, *x*, applies to the entire line.

Existential Instantiation (EI): Just as we have to be careful about generalizing to universally quantified statements, so also we have to be careful about instantiating an existential statement. When you instantiate an existential statement, you cannot choose a name that is already in use. That's because we are not justified in assuming that the individual constant is the same from one instantiation to another. So, if you have to instantiate a universal statement and an existential statement, instantiate the existential first. The universal instantiation can then assert the same constant as the existential instantiation, because there are no restrictions on **UI**.

C. Change of Quantifier (CQ)

You can infer existential statements from universal statements, and vice versa, without having to instantiate first. In ordinary language, the phrase equivalences are as follows:

> "All are" is equivalent to "It's not the case that there is one that is not."
> "It is not the case that all are not" is equivalent to "Some are."
> "Not all are" is equivalent to "Some are not."
> "It is not the case that there is one" is equivalent to "None are."

$(x)Sx :: \sim(\exists x)\sim Sx$
$\sim(x)\sim Sx :: (\exists x)Sx$
$\sim(x)Sx :: (\exists x)\sim Sx$
$(x)\sim Sx :: \sim(\exists x)Sx$

D. Conditional and Indirect Proof

In predicate logic, conditional and indirect proof follow the same structure as in propositional logic:

1. Assume a statement, P, derive another statement, Q, then discharge the assumptive proof by asserting, $P \supset Q$.
2. Assume a statement, $\sim P$, derive a contradiction, then discharge the assumptive proof by asserting, P.

In predicate logic, however, there is one restriction on **UG** in an assumptive proof: when the assumption is a free variable, **UG** is not allowed from the line where the free variable occurs. That is because the assumption **names an individual** assumed to have the property designated by the predicate. It does not, therefore, act as **an arbitrary individual** from which we may generalize to a universal statement.

E. Demonstrating invalidity

There are two methods to demonstrate that a predicate logic argument is invalid: Counterexample Method and Finite Universe Method.

The **counterexample method** follows the same steps as are used in Chapter 1: replace the premises with another set we know to be true; replace the conclusion with one we know to be false. Each replacement must follow the same form as the original:

All oranges are fruits.
<u>Some vegetables are not fruits.</u>
Some oranges are not vegetables.

$(x)(Ox \supset Fx)$
$\underline{(\exists x)(Vx \bullet {\sim}Fx)}$
$(\exists x)(Ox \bullet {\sim}Vx)$

All citizens are eligible to vote.
<u>Some people are not eligible to vote.</u>
Some citizens are not people.

The **finite universe method** enlists indirect truth tables to show, truth-functionally, that a predicate logic argument is invalid:

1. Suppose a universe that contains only one member.
2. Instantiate the premises and conclusion to the same constant.
3. Construct an indirect truth table to determine whether or not the argument is invalid.
4. If the argument does not prove invalid with a single-member universe, try two members.

Note: if you do not prove the argument is invalid assuming a three-member universe, double-check your work and then consider using the inference rules to construct a proof. It may be that the argument is, in fact, valid.

F. Relational Predicates

Predicate logic notation allows us to work with **relational predicates** (two or more place predicates), rather than only single-place predicates:

Joe is to the right of Stew:	Rjs
Joe and Stew like each other:	$Ljs \bullet Lsj$
Everyone likes someone:	$(x)(Px \supset (\exists y)Lxy)$

Relational predicates include a number of different types:

- **Symmetrical relationship**: Can be illustrated by the following: If A is married to B, then B is married to A.
- **Asymmetrical relationship**: Can be illustrated by the following. If A is the father of B, then B is *not* the father of A.

- **Nonsymmetrical**: When a relationship is neither symmetrical nor asymmetrical. For example: If Kris loves Morgan, then Morgan may or may not love Kris.
- **Transitive relationship**: Can be illustrated by the following: If A is taller than B, and B is taller than C, then A is taller than C.
- **Intransitive relationship**: Can be illustrated by the following: If A is the mother of B, and B is the mother of C, then A is *not* the mother of C.
- **Nontransitive relationship**: Can be illustrated by the following: If Kris loves Morgan and Morgan loves Terry, then Kris may or may not love Terry.

Proofs involving relational predicates require an additional restriction on **UG**:

- Universal generalization cannot be used if the instantial variable is free in any line that was obtained by existential instantiation (EI).

G. Identity

Identity is a two-way relation holding between a thing and itself.

The symbolic notation for identity statements is the use of =. So, "Fifty Cent is Curtis Jackson," becomes $f = c$. When we deny identity, we use ≠. So, "Fifty Cent is not Marshall Mather," becomes $f \neq m$.

When we want to distinguish between members of a class, but the statement we assert does not specify names, we can use the identity symbol to help. "At least two dogs are in the park," becomes $(\exists x)(\exists y)(Dx \bullet Dy \bullet Px \bullet Py \bullet x \neq y)$. Similarly, when we want to assert an exact number, but we do not specify names, we use the identity symbol. "There is exactly one dog in the park," becomes "$(\exists x)(Dx \bullet Px \bullet (y)[(Dy \bullet Py) \supset x = y)$.

The following are special kinds of identity relations:

- **Reflexive property**: The idea that *anything is identical to itself*.
- **Irreflexive relationship**: Can be illustrated by the following expression: "Nothing can be taller than itself."
- **Nonreflexive**: When a relationship is neither reflexive nor irreflexive.

Proofs involving the identity relation require an additional three special rules:

- Rule 1 expresses the reflexive property (anything is identical to itself).
- Rule 2 is a replacement rule (a = b can be replaced with b = a, or a ≠ b with b ≠ a).
- Rule 3 is a special case of the transitive property (if a = b and b = c, then a = c).