Baronett, *Logic* (4th ed.)
Chapter Guide

Chapter 7: Propositional Logic

The basic components of **propositional logic** are statements.

A. Logical Operators and Translations

There are two types of statement: simple and complex, or compound.

A **simple** statement is one that does not contain another statement as a component. These statements are represented by capital letters A to Z.

A **compound** statement contains at least one simple statement as a component, along with a **logical operator**, or *connective*.

There are five compound types represented by logical operators:

Compound Type	**Negation**	**Conjunction**	**Disjunction**	**Conditional**	**Biconditional**
Operator	~	•	v	⊃	≡
Operator Name	Tilde	Dot	Wedge	Horseshoe	Triple Bar

B. Compound Statements

Here are four rules to help organize your thinking about translating complex statements:

1. Remember that every operator *except* the negation is always placed in between statements.
2. Tilde is always placed to the left of whatever is to be negated.
3. Tilde never goes by itself in between two statements.
4. Parentheses, brackets, and braces are required to eliminate ambiguity in a complex statement.

Following these rules assures that a compound statement is a **well-formed formula**.

There is always only one **main operator** in any statement, and that operator is either one of the four that appear *in between* statements or the tilde that appears *in front* of the statement that is negated.

C. Truth Functions

Every simple statement has a **truth value**: it is either true or false.

The truth value of a compound statement is uniquely determined by the meaning of the five operators and the truth values of its component propositions. Any such proposition is called a **truth-functional proposition**, or a truth function.

Negation tells us "it is not the case that …"

Negation:

p	$\sim p$
T	F
F	T

Conjunction tells us "both … are the case." Conjunctions are only true when both conjuncts are true.

Conjunction:

p	q	$p \bullet q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction tells us "at least one is the case …" Disjunctions are only false when both disjuncts are false.

Disjunction:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

A **conditional statement** mirrors the concept of validity: If the antecedent is true, the consequent cannot be false. Thus, a conditional statement is only false when a true antecedent implies a false consequent.

Conditional:

p	q	$p \supset q$
T	T	T
T	F	F

F	T	T
F	F	T

The **biconditional** tells us "either both are the case, or neither is …" Thus, a biconditional statement is true when both statements are true, or both are false.

Biconditional:

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

D. Truth Tables for Propositions

When we construct a **truth table** to determine the possible truth values of a given statement, it is important to know:

1. That the number of **simple statements** in the compound statement determines the number of **rows** in the truth table. For example, a compound statement with two simple statements requires a four-row truth table. A compound statement with three simple statements requires an eight-row truth table.
2. The **main operator is calculated last** in the order of operations since it determines the final truth values of the statement.

E. Contingent and Noncontingent Statements

A **contingent** statement is true on at least one row and false on at least one row in the main operator column (it is sometimes true, sometimes false).

A **noncontingent** statement is *not* dependent on the truth values of the component parts. Such a statement is either always true (a **tautology**) or always false (**self-contradictory**).

F. Logical Equivalence, Contradictory, Consistent, and Inconsistent Statements

Two or more statements are **logically equivalent** when they have identical truth tables under the main operator on every line of their respective truth tables.

Two statements are **contradictory** when they have opposite truth values under the main operator on every line of their respective truth tables.

Two or more statements are **consistent** when they have at least one line on their respective truth tables where the main operators are true.

Two or more statements are **inconsistent** when they do not have even one line on their respective truth tables where the main operators are both true.

G. Truth Tables for Arguments

Truth tables can also be constructed for arguments to determine validity or invalidity by calculating the truth values under each premise and the conclusion.

When there is even one row of the truth table on which the premises are all true while the conclusion is false, the argument is *invalid*. A truth table in which there is not even one row of the argument on which the premises are all true while the conclusion is false is *valid*.

H. Indirect Truth Tables

The **indirect truth table** is a powerful and expedient tool for testing an argument's validity:

1. Assume the premises are true and the conclusion is false. In other words, assume the argument is invalid by placing a T under the main operator of each of the premises and an F under the main operator of the conclusion.
2. Calculate backwards from the assumed value to the statements' constituent elements.
3. If you can calculate all of the elements without contradicting the truth definitions for any given operator, the argument is *invalid*.
4. If you cannot calculate all of the elements without contradicting the truth definitions for the given operator, the argument is *valid*.