Integration II

Separating the variables and integrating with limits

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Answers to additional problems

The $-H$ terms has been placed outside the integral because it is a constant. This is a a good assumption provided the temperature range is limited.

20.1 We first separate the variables, $\int_{G_1/T_1}^{G_2/T_2} 1 \times \partial(G/T) = -H\int_{T_1}^{T_2} \frac{1}{T^2}\,\partial T$

We obtain, $\left[\frac{G}{T}\right]_{G_1/T_1}^{G_2/T_2} = +H\left[\frac{1}{T}\right]_{T_1}^{T_2}$

We then insert the limits, $\frac{G_2}{T_2} - \frac{G_1}{T_1} = H\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$

We often call this last expression the **Gibbs–Helmholtz equation** and then express it in a slightly different form, as $\frac{\Delta G_2}{T_2} - \frac{\Delta G_1}{T_1} = \Delta H\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$

20.2 First we rewrite the integral, $w = -\int_{V_\text{initial}}^{V_\text{final}} p_\text{ext}\;\mathrm{d}V$

Integrating yields, $w = -p_\text{ext}\,[V]_{V_\text{initial}}^{V_\text{final}}$

Inserting the variables yields, $w = -p_\text{ext}(V_\text{final} - V_\text{initial})$

so $w = -p_\text{ext}\,\Delta V$

20.3 We first separate the variables, $I \times \mathrm{d}t = \mathrm{d}Q$

Writing the equation as an integration problem, $\int I\;\mathrm{d}t = \int 1\,\mathrm{d}Q$

Integrating, $I\,t = Q + c$

20.4 We first separate the variables, $\frac{1}{1} \times \mathrm{d}[\mathrm{A}] = -k\,[\mathrm{A}]^2\,\mathrm{d}t$

then rearrange, $\frac{1}{[\mathrm{A}]^2}\,\mathrm{d}[\mathrm{A}] = -k\,\mathrm{d}t$

We then write as an integration problem, $\int_{[\mathrm{A}]_0}^{[\mathrm{A}]_t} \frac{1}{[\mathrm{A}]^2}\,\mathrm{d}[\mathrm{A}] = \int_0^t -k\,\mathrm{d}t$

Integration yields, $\left[-\frac{1}{[\mathrm{A}]}\right]_{[\mathrm{A}]_0}^{[\mathrm{A}]_t} = -k\left[t\right]_0^t$

Substituting in $-\frac{1}{[\mathrm{A}]_t} - -\frac{1}{[\mathrm{A}]_0} = -k(t-0)$

Simplifying $\frac{1}{[\mathrm{A}]_t} = kt + \frac{1}{[\mathrm{A}]_0}$

20.5 Separating the variables, $a\,\mathrm{d}t = \mathrm{d}v$

Writing the problem as an integration, $\int_0^t a\,\mathrm{d}t = \int_u^v 1\,\mathrm{d}v$

We perform the integration, $[at]_0^t = [v]_u^v$

We insert the variables, $[at - a \times 0] = [v - u]$

And finally, we tidy up, $at = v - u.$

We usually express this equation is, $v = u + at$ or $a = \dfrac{v-u}{t}$

20.6 We rewrite the expression, recalling from Chapter 9 how we can express a third root as a power of ⅓, $\int_2^4 \left[5^{1/3}\,x^{1/3}\right]\mathrm{d}x.$

We position the factors of $5^{1/3}$ and ¾ outside the bracket because they are constants.

We integrate the expression, $5^{1/3}\left[\dfrac{3}{4}x^{4/3}\right]_2^4 = 5^{1/3} \times \dfrac{3}{4}\left[x^{4/3}\right]_2^4$

We insert the limits, $5^{1/3} \times \dfrac{3}{4}(4^{4/3} - 2^{4/3})$

The respective values are, $5^{1/3} = 1.710$ $\quad 4^{4/3} = 6.350$ $\quad 2^{4/3} = 2.520.$

Therefore, the value of the integral = 1.710 × 0.75 (6.350 – 2.520) = 4.91.

20.7 We first rewrite the equation slightly, $\dfrac{\mathrm{d}\mu}{\mathrm{d}T} = kT^{1/2}$

and we separate the variables, $\mathrm{d}\mu = kT^{1/2}\,\mathrm{d}T$

We write it as an integration problem, $\int 1\,\mathrm{d}\mu = \int kT^{1/2}\,\mathrm{d}T$

We integrate, $\mu = \tfrac{2}{3}kT^{3/2} + c$

- We insert the 1 as a mathematical dodge thereby clarifying that the dμ operator is actually working upon something.

20.8 We separate the variables, $v\,\mathrm{d}t = \mathrm{d}\ell$

We write as an integration problem, $\int v\;\mathrm{d}t = \int 1\,\mathrm{d}\ell$

We integrate the expression, $vt = \ell + c$

We can rephrase this equation as, $v = \dfrac{\text{distance covered}}{\text{time elapsed}}$

20.9 We first separate the variables, $\mathrm{d}[\mathrm{A}] = -k[\mathrm{A}]\,\mathrm{d}t$

We rearrange, $\dfrac{1}{[\mathrm{A}]}\mathrm{d}[\mathrm{A}] = -k\,\mathrm{d}t$

At $t = 0$, the concentration of A is $[A]_0$, whereas at time t the concentration is $[A]_t$. We then write as an integration problem, $\int_{[A]_0}^{[A]_t} \frac{1}{[A]} \, d[A] = \int_0^t -k \, dt$

and integrate as, $\left[\ln[A]\right]_{[A]_0}^{[A]_t} = -k[t]_0^t$

which can be written out as $\ln[A]_t - \ln[A]_0 = -k(t-0)$

and simplified as $\ln\frac{[A]_t}{[A]_0} = -kt$

or $\ln[A]_t = -kt + \ln[A]_0$

or $[A]_t = [A]_0 \exp(-kt)$

20.10 We integrate the function, $\left[-\frac{1}{4}\cos 4x\right]_{\pi/12}^{\pi/6}$

We insert the limits, $\left(-\frac{1}{4}\cos\left(\frac{4\pi}{6}\right)\right) - \left(-\frac{1}{4}\cos\left(\frac{4\pi}{12}\right)\right)$

For ease, we factorize, taking out the common factor of $-\frac{1}{4}$, and simplify the arguments of the cosines, $-\frac{1}{4}\left[\cos\left(\frac{2\pi}{3}\right) - \cos\left(\frac{\pi}{3}\right)\right]$

2π rad = 360°, so $\pi/6$ rad = 30°.

We insert the limits, $-\frac{1}{4}[(-0.5) - (0.5)]$

we see the value of the integral is, $-\frac{1}{4} \times (-1) = \frac{1}{4}$.