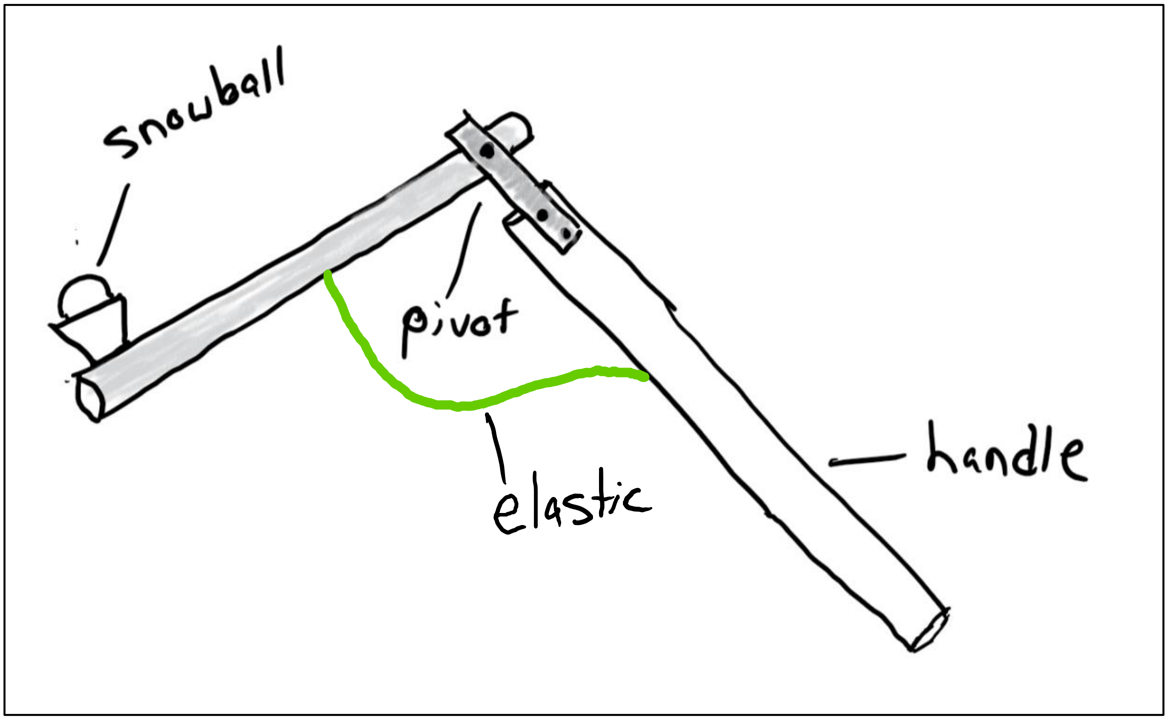
Snowball Thrower Performance Report ~ Dynamics 2340

B. MacCallum April 4th, 2017

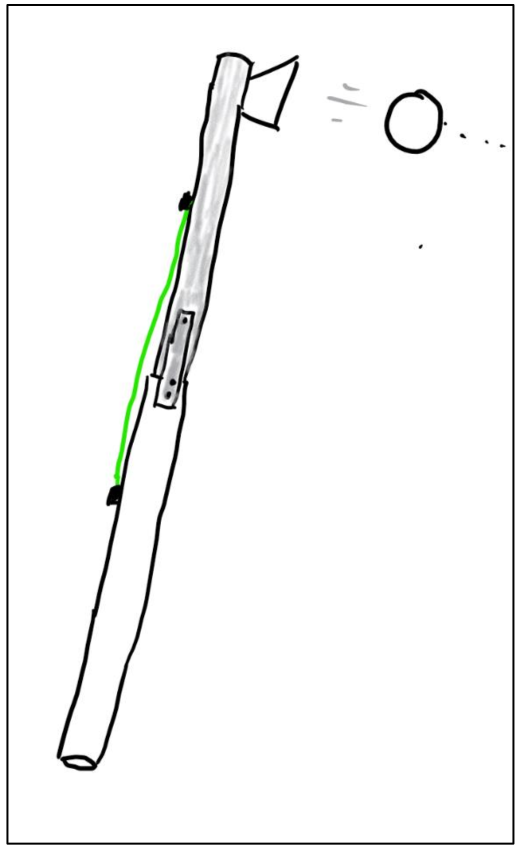
Design Overview

We designed a snowball thrower that takes advantage of human power, while optimizing the potential distance by extending the user's effective arm length with 2 articulated segments joined at a pivot point. This simple mechanism mimics the movement of a human arm in hopes of using the same dynamic properties that are proven to have considerable accuracy and distance potential when properly calibrated.



The device is built from 2 segments of PVC pipe, attached at their ends by a connecting piece that offers a pivot point free of obstruction. The larger of the 2 pieces acts as a handle, which is where the user will have contact when propelling the snowball into flight. The 2nd segment of pipe is slightly narrower, with a length that is roughly equal to the first. On this piece, the snowball is affixed in a cup that will hold it in place until the release point. An elastic is connected between these two pivoting arms, which provides a linearly increasing amount of force against rotation throughout the throw. This is a means to have the ball naturally release at a point in which the cup where the snowball is held is travelling at a speed that is slower than the ball. Much like the muscles in an elbow that start to resist motion in the latter part of a throw to slow down the forearm, this elastic prevents a sharp impulse on the device when the pivoting arm is forced to a stop. This elastic has multiple possible solutions to hook onto on both arms, allowing for calibration to the tension to optimize the performance of the thrower.

Shown in the figure to the right is the new state of the device at the point when the pivoting arm becomes quasi-parallel with the handle arm. In theory, the elastic will completely stop rotation once the arm has reached this state, resulting in a maximized radius that is the sum of the 2 arms' lengths. Adjusting the position of the elastic results in a significant change in how much force by the user is required to impose a linear form shown in the sketch. This parameter, therefore, is an important factor in the device's success, taking into consideration the abilities of the user.



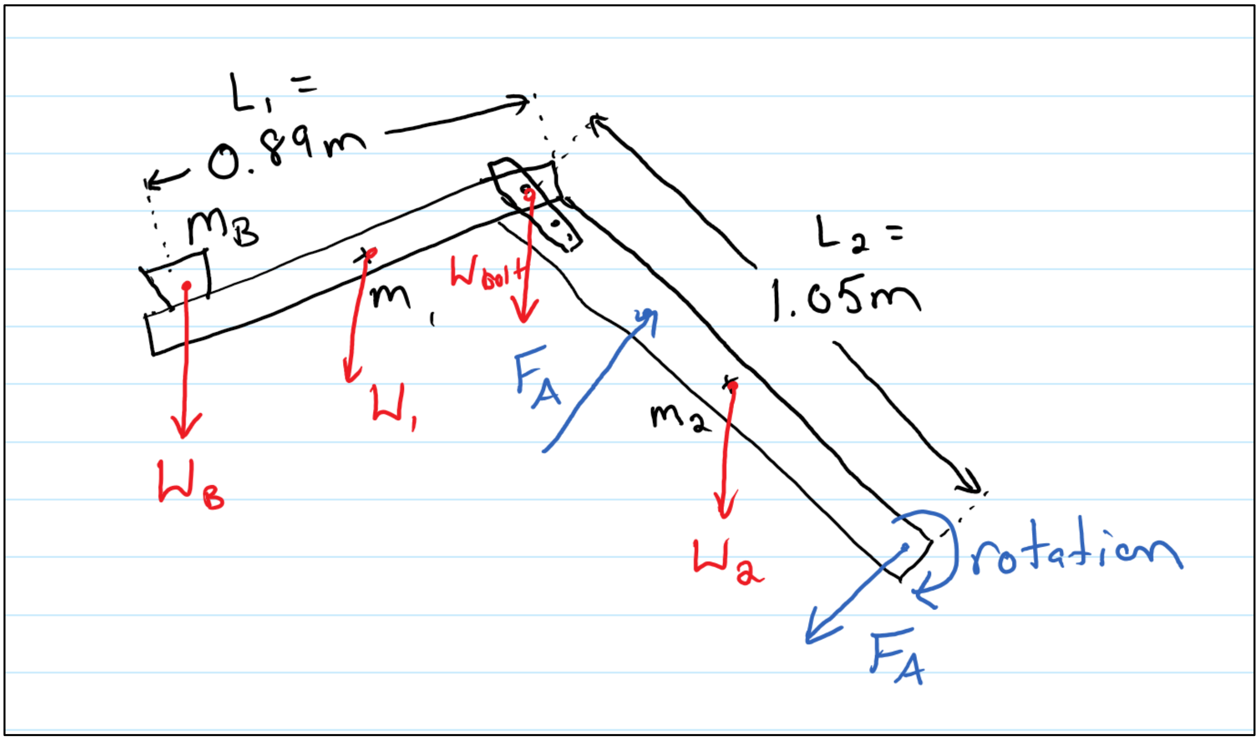
A typical analysis for a trebuchet thrower could be applied to this device with an acceptable level of accuracy. A trebuchet uses a slack string fixed to a lever arm, rather than a system of two rigid arms; an ideal trebuchet implies the string is under constant tension (never taut) throughout the duration of the throw. For this reason, a similar process of analyzing this device may be carried out to determine what dimensions, weights, and forces will maximize the performance in real-world testing such as the competition that this device has been designed for. An obvious exception is the implementation of a counteracting force in the elastic, as well as a non-weightless pivot arm, unlike a nearly weightless string used in a trebuchet.

Results of previous analysis as well as research into trebuchet models helped to realize an estimation for the ideal dimensions of each arm, as shown in the photo below. Further analysis will verify these initial measurements and help to further explain the performance of the device.

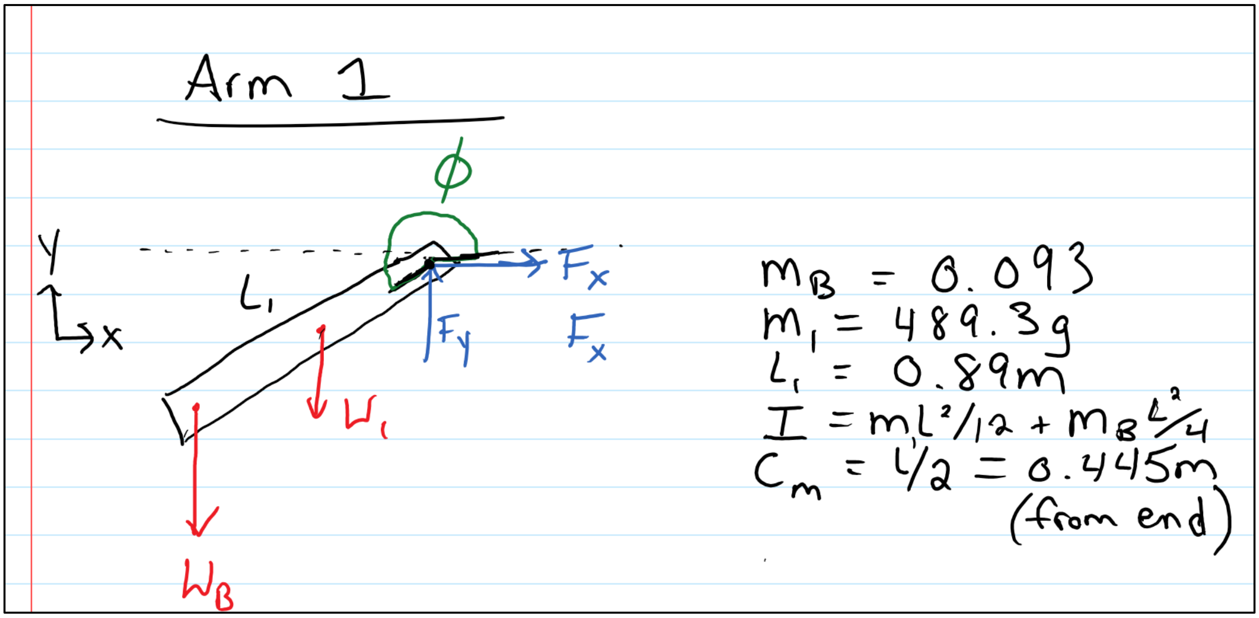
Analysis

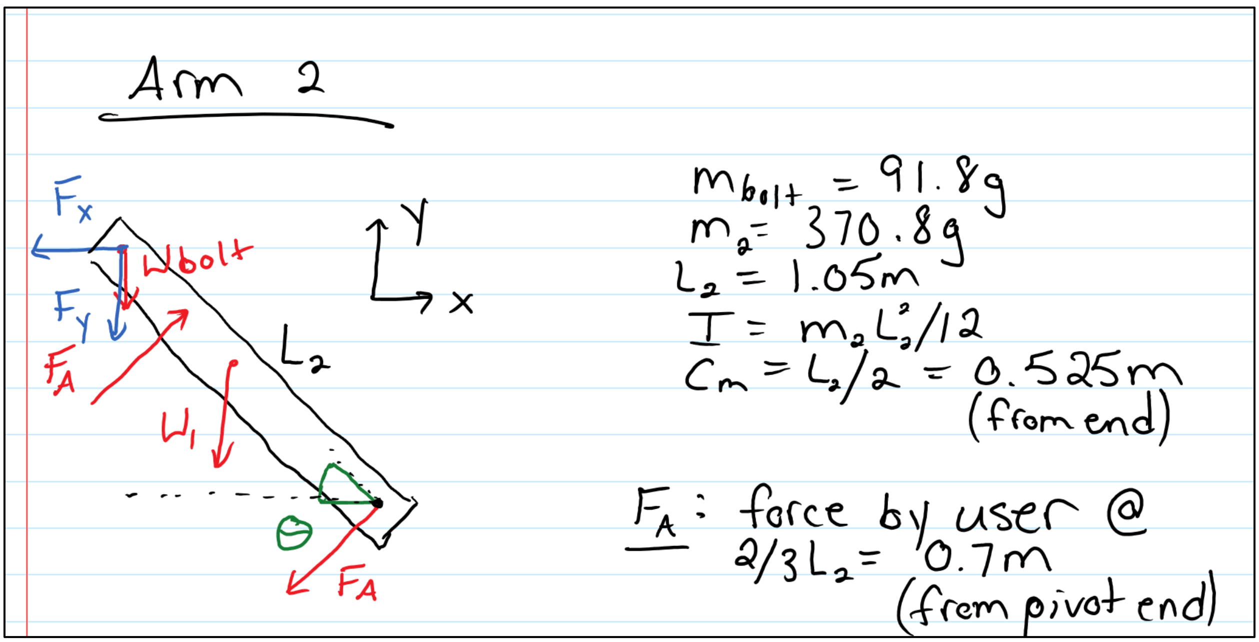
The key focus of the analysis conducted on the snowball thrower is the speed of the snowball just before it is released into its trajectory. Each arm of the thrower is analyzed individually to determine its rate of rotation at the point in which the ball is released, giving a solution to the ball's speed.

This free-body diagram includes the entire thrower, showing all external forces. F_A is the force exerted by the user at the 2 points in which the hands contact the handle. The thrower is assumed to be rotating about the very end of the handle. The elastic that resists the rotation of arm 1 is not included in the study of the ball's speed. The elastic is used as a method of calibrating the release point of the ball



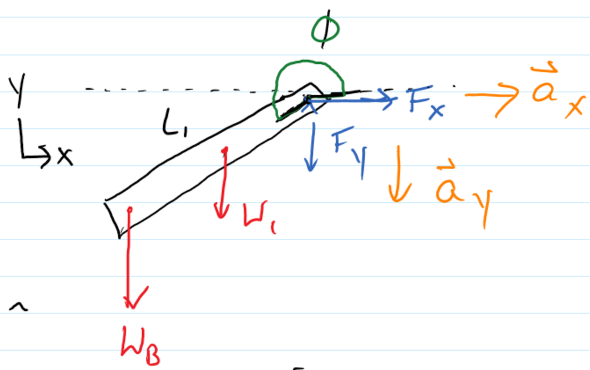
While this diagram is a good representation of all the forces that must be taken into consideration, studying the system is not the most effective method of analyzing the movement of each arm. Shown below are free-body diagrams of each individual arm. The connection point of the two arms is denoted by resultant forces F_x and F_y. Arm 1's angle of rotation is "phi" and arm 2's rotation is denoted by "theta". The rate of rotation of arm 1 is dependant on that of arm 2. This is taken into account when solving for F_x and F_y, which is proportional to the acceleration of the point in which the two arms join. This acceleration is directed toward the centre of rotation of arm 2, with magnitude $(\dot{\theta})^2 \cdot L_2$.

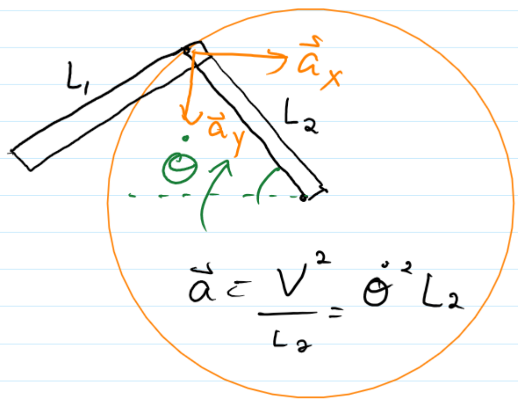




Shown below is a detailed focus on arm 1, who's rate of rotation is directly related to the speed of the ball at release. The forces F_x and F_y are solved for in this next step, which makes it possible to determine the net moment on the arm about the end which holds the ball. While the angle "phi" is shown about the opposite end, the rate of rotation about each end with respect to a fixed coordinate system is equivalent.

Arm 1





$$a_x = \vec{a}\cos\theta$$
$$a_y = -\vec{a}\sin\theta$$

$$\boxed{\begin{aligned} a_x &= \dot{\theta}^2 L_2 \cos\theta \\ a_y &= -\dot{\theta}^2 L_2 \sin\theta \end{aligned}}$$

$$\Sigma F_x = F_x = (\dot{\theta}^2 L_2 \cos\theta)(m_1 + m_b)$$
$$\Sigma F_y = -F_y - W_B - W_1 = (-\dot{\theta}^2 L_2 \sin\theta)(m_1 + m_b)$$
$$F_y = -(m_b + m_1)g + (\dot{\theta}^2 L_2 \sin\theta)(m_1 + m_b)$$
$$F_y = (m_b + m_1)(\dot{\theta}^2 L_2 \sin\theta - g)$$

The net moment is integrated with respect to the angle “phi” and is set equal to the moment of inertia multiplied by the angle phi integrated with respect to itself.

$$\Sigma M_B = -\left(\frac{W_1}{2} + F_y\right) L_1 \cos\phi - F_x L_1 \sin\phi$$
$$\int M\, d\phi = \frac{I\dot{\phi}^2}{2} \qquad I = \frac{m_1 L_1^2}{3} \text{ (about } m_b\text{)}$$
$$-\left(\frac{W_1}{2} + F_y\right) L_1 \sin\phi + F_x L_1 \cos\phi = \left(\frac{m_1 L_1^2}{6}\right)\dot{\phi}^2$$

After substituting the values for F_y and F_x, this equation can be rearranged to solve for the rate of rotation of arm 1, “phi dot”.

$$\dot{\phi} \quad \sqrt{\frac{-\frac{g}{2}\sin\phi + \left(\frac{m_b}{m_1}\right)\dot{\theta}^2 L_2\left(\cos(\theta-\phi)\right)}{L_1/6}}$$

Shown below is the equation for “phi dot” after all known values are substituted into the equation. The speed of the ball at release is then calculated from the rate of rotation. Theta dot can not be solved for directly, as the integral of the net moment on arm 2 is not possible to solve. Numerical analysis shall be carried out to estimate the speed of the snowball upon release.

$$\dot{\phi} = \sqrt{6.74\left(-\frac{g}{2}\sin\phi + 0.1995(\dot{\theta})^2\cos(\theta-\phi)\right)}$$

$$\vec{V} = L_1\dot{\phi} = \sqrt{5.34\left(-\frac{g}{2}\sin\phi + 0.1995(\dot{\theta})^2\cos(\theta-\phi)\right)}$$

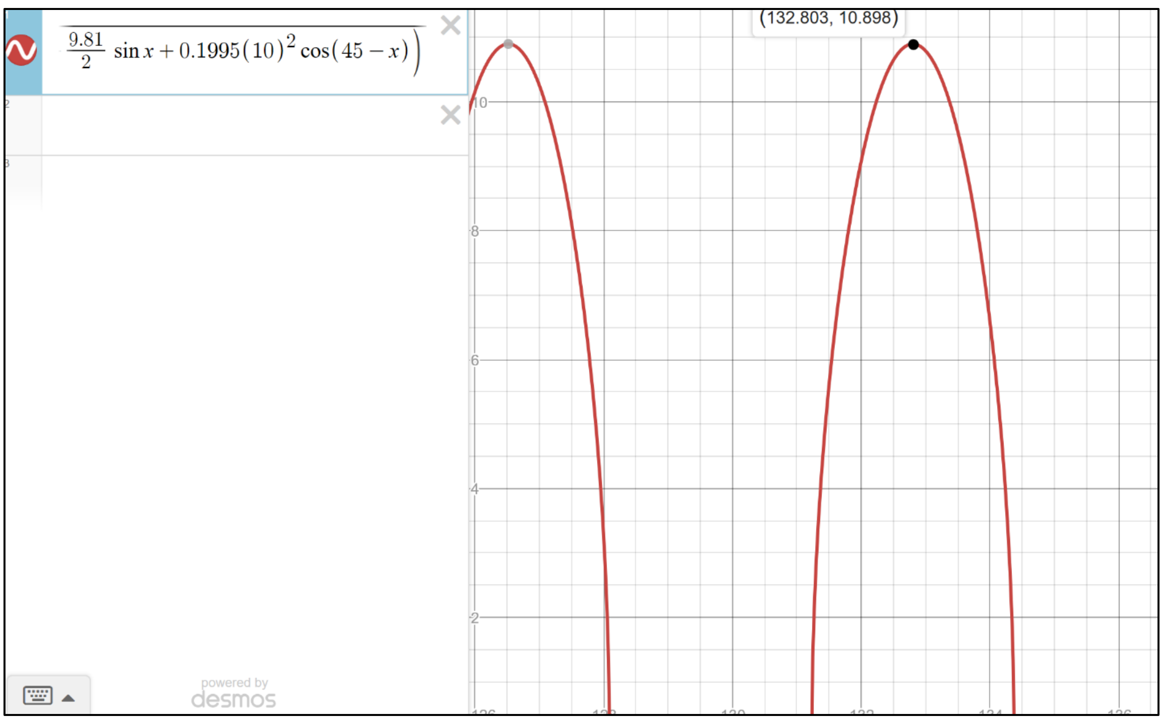
To have the speed equal a real number (non-imaginary), phi must be within a certain limit for each given theta dot value. The values for phi shown in the table to the right are the maximum possible angles (to one decimal place) before the speed no longer equals a real number. An assumption is made that the angle theta will be equal to 45 degrees when the ball is released. When the two arms are perfectly parallel, phi shall be 90 degrees greater than theta. In a scenario in which the ball releases and the two arms are linear, the angle phi should be 135 degrees. As seen in the table, phi approaches 135 but cannot reach it while the speed remains a real number. This concludes that it is not possible for the two arms to be linear at any point within the given range of “theta dot”. The greater the rate of rotation of arm 2, the closer to 135 degrees phi becomes.

Theta dot	Theta	Phi	Speed
1	45	131.9	1.320719
2	45	132	1.018571
3	45	132.2	0.600208
4	45	132.6	0.093063
5	45	133.2	1.008304
6	45	133.8	0.757336
7	45	134.1	0.37502
8	45	134.2	1.453413
9	45	134.3	1.243825
10	45	134.3	2.503563
11	45	134.3	3.389023
12	45	134.4	2.016075
13	45	134.4	2.76757

If the same table is manipulated so that the angle phi remains constant with an increasing theta dot, we see a much greater possible speed for the snowball. It is estimated that theta dot is in the range of 9-11 rad/s by analyzing the video data of the throws. This would return a speed of the snowball of about **7m/s**, launched at 41.9 degrees above the horizontal.

Shown below is a plot of speed versus phi. The graph is only continuous over small intervals that repeat infinitely. Therefore, it is theoretically impossible to have phi equal 135 degrees, as it lies in an area of discontinuity while theta is 45 degrees.

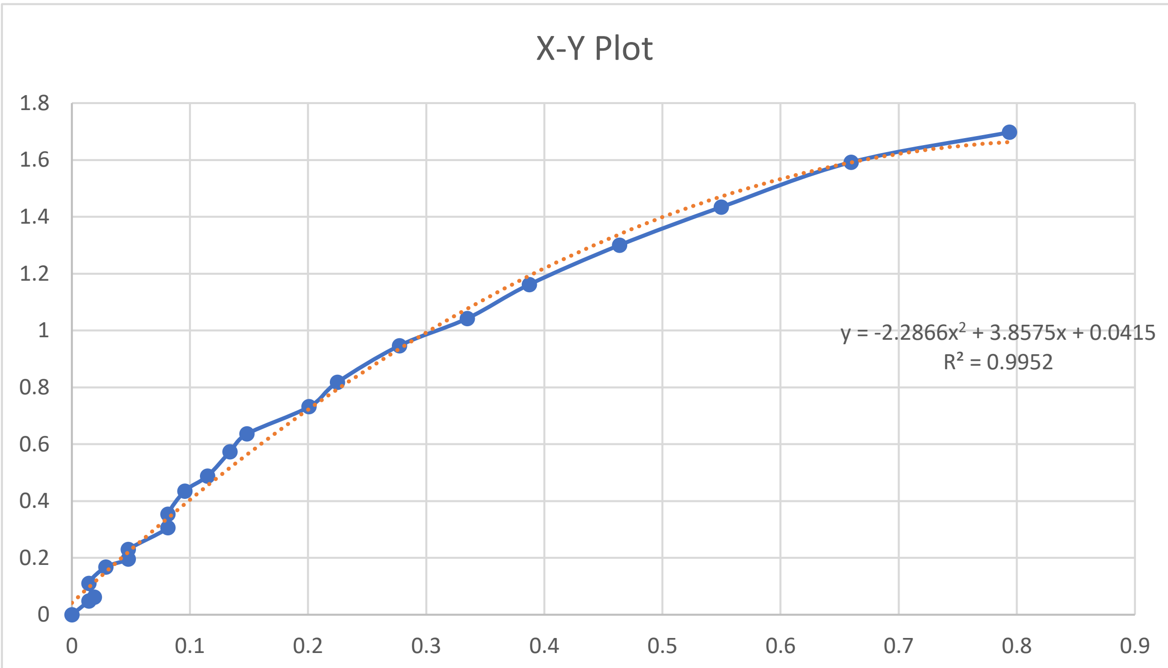
Theta dot	Theta	Phi	Speed
1	45	131.9	1.320719
2	45	131.9	1.814911
3	45	131.9	2.424165
4	45	131.9	3.080961
5	45	131.9	3.76047
6	45	131.9	4.452305
7	45	131.9	5.151502
8	45	131.9	5.855425
9	45	131.9	6.562552
10	45	131.9	7.27195
11	45	131.9	7.983012
12	45	131.9	8.695332
13	45	131.9	9.408622



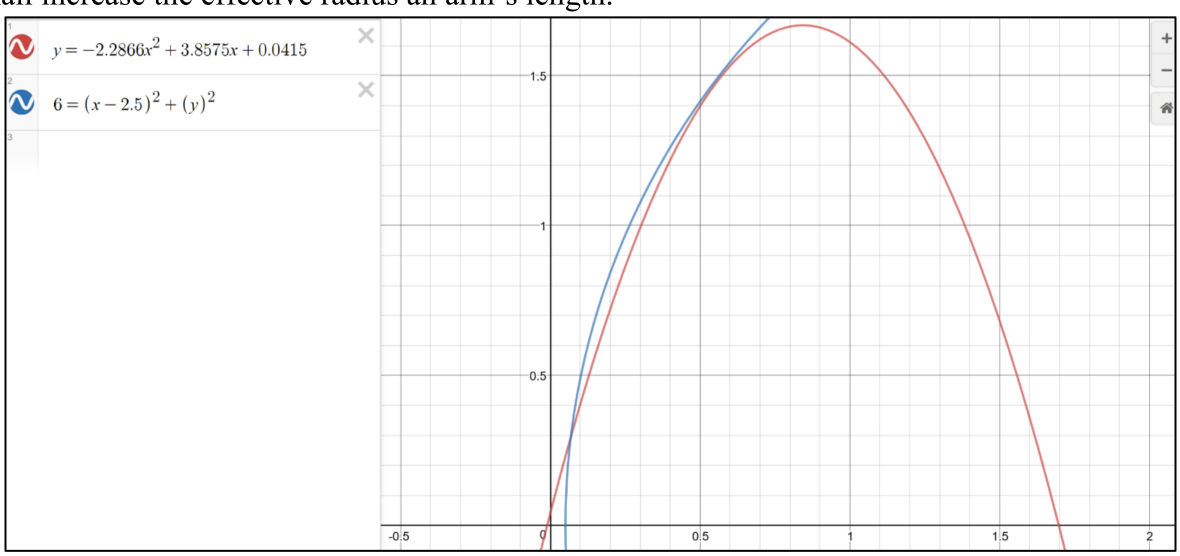
As stated previously, this thrower may be compared to a trebuchet, which has been previously analyzed. It was found that a trebuchet with similar dimensions has a maximum speed of about **4m/s**. Unlike the trebuchet, this thrower takes advantage of human force, which varies depending on the user, yet significantly outweighs the force applied by a mass attached to the lever arm. For this reason, we see a significant increase in potential launch speed with this articulated arm thrower.

Data & Results

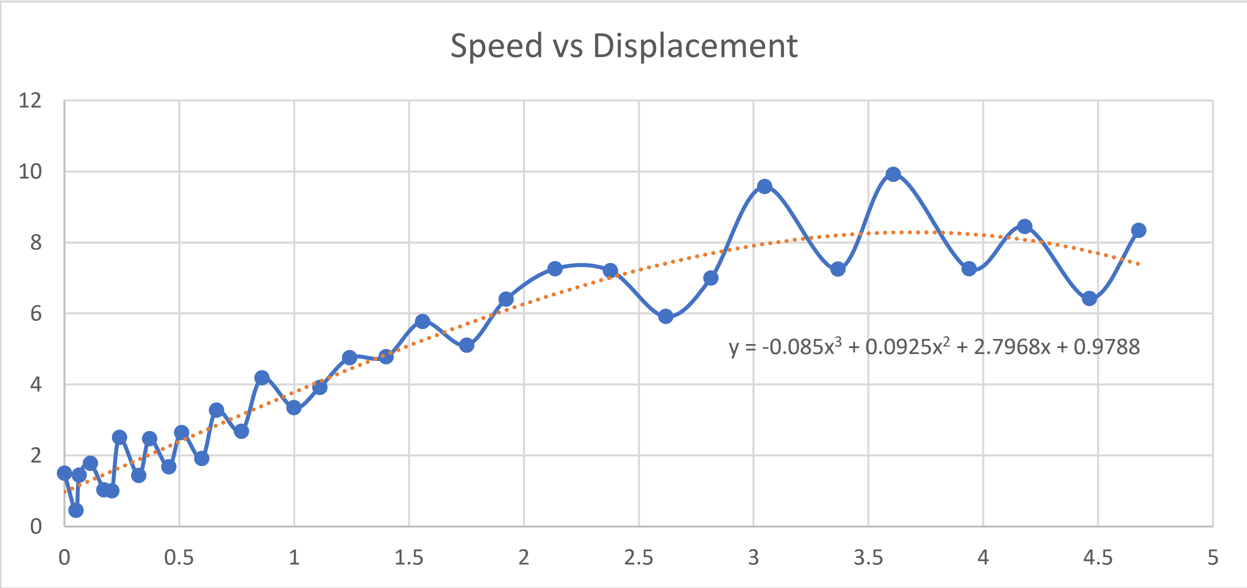
Shown below is a series of plots created from the video recording while testing our thrower. The plot of the ball's motion closely resembles a parabolic shape with an equation shown in the graph below.



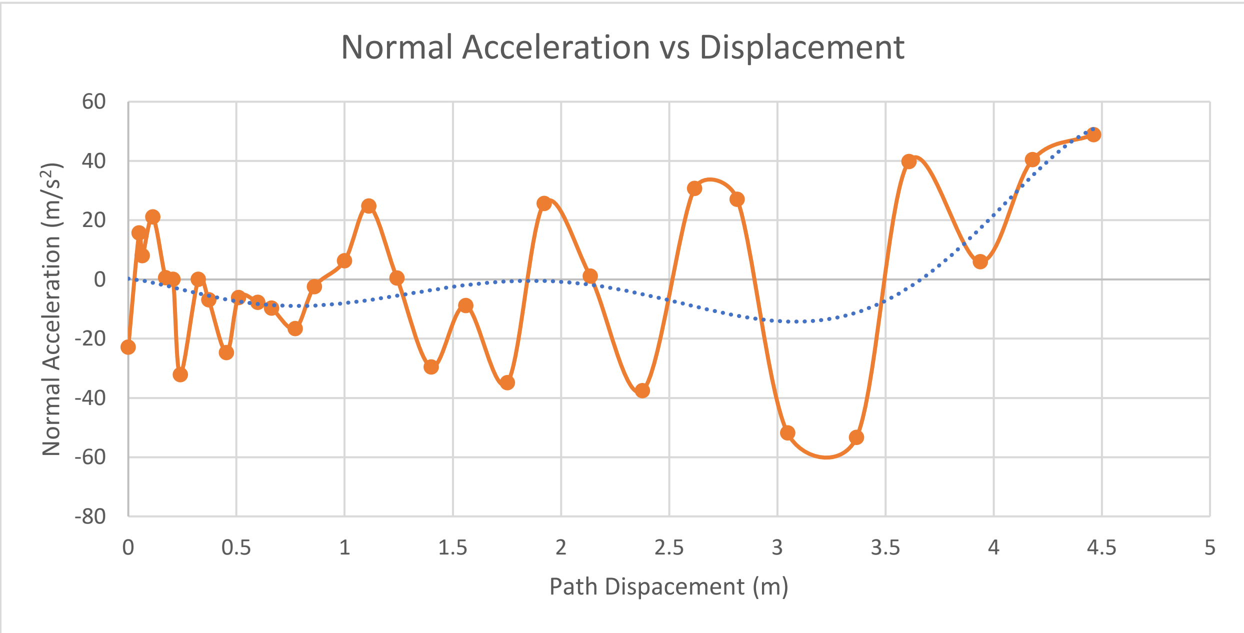
The parabolic function that is fitting to the ball's motion is graphed below, alongside a plot of a circular function that most closely relates to its counterpart. This circular function has a radius of approximately 2.5m. Knowing the dimensions of the thrower arm, it should have a maximum radius of rotation of 1.94m. It may be concluded that the user's arms act in the rotation, which shall increase the effective radius an arm's length.



The speed of the ball is shown to peak at approximately **8m/s**. This can be assumed to be the point of release, at it is expected to slow down during its trajectory with consideration for air resistance and the gravitational force counteracting the ball's vertical motion.

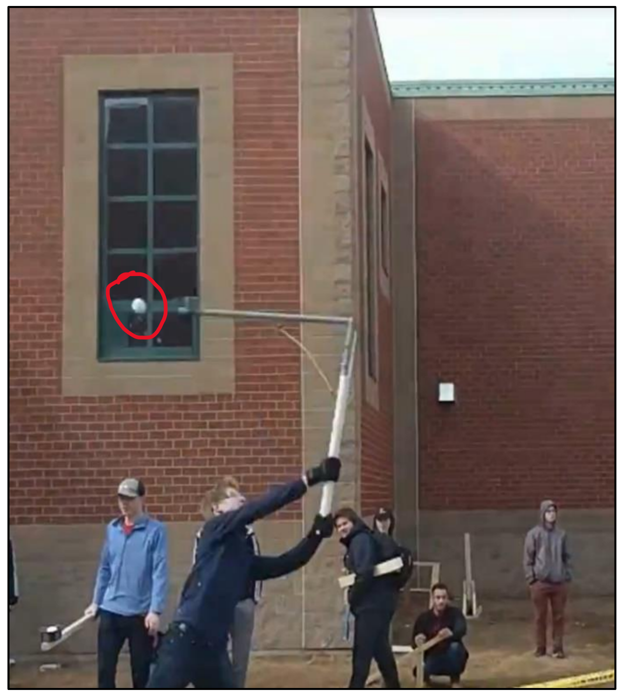


Normal acceleration is caused by rotational motion about some centre pivot point. The magnitude of the normal acceleration of the snowball is dependant on its speed and radius of rotation. The plot below shows the normal acceleration of the snowball versus displacement along the path. Using a polynomial of best fit, it is apparent that the magnitude fluctuates throughout the period when the ball is in contact with the thrower. At about 3.7m, which happens to be the point in which the speed of the ball peaks, the normal acceleration changes signs and the magnitude peaks.



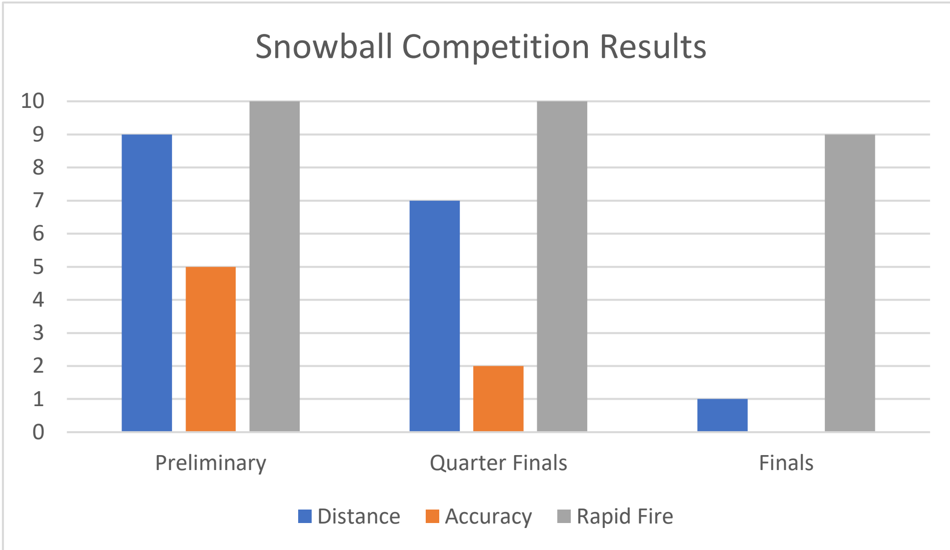
Competition

During the competition, our thrower was extremely inconsistent in its performance, despite the analysis showing that the ball would have a pleasing release speed. Shown in the photos below are two examples of throws during the competition. The first is a successful throw that launched the ball a considerable distance. The second was a failed throw that resulted in the ball landing behind the thrower. It is apparent that is the failed throw, the ball releases from the cup far too early, resulting in a launch angle that is not favourable to its range. This does not disprove the analysis that states the device would have a launch speed of 7m/s, but instead takes into question the techniques used to control the release point of the ball.



Because of this uncertainty in the release of the ball, we say a variety of results in the distance competition. Two trials had satisfying results, while the final round did not. The accuracy of the device also did not hold up well during the competition. Unrepeatability of the user's exact movements made it extremely difficult to have three consecutive throws hit a target point with precision. Finally, rapid fire was the most successful competition for our device. We were able to easily load snowballs onto the thrower and repeat the launch motion repeatedly.

The graph below is a representation of the thrower's outcome in each of the categories at each stage of the competition. The results are shown on a scale of 1-10, based off personal judgement of the outcome. The first round was the most successful, with each following round having diminishing results. Our group was eliminated from the accuracy category before the finals, hence the non-inclusion of this in the graph.



Discussion & Improvements

Unlike previous analyses that were completed on our original design concept, a trebuchet thrower, the analysis for this device considered the weights of the device itself; the mass of the plastic tubing, metal bolts, and even the cup holding the ball are too significant to ignore. After testing the device repeatedly and studying the video data, there seems to be a certain level of accuracy in the analysis' prediction. Unfortunately, I was unable to measure the exact force one of our group members could apply, as well as consider the exact motion of the real device. For this reason, several assumptions were necessary to proceed, and produced a result that could be loosely related to what we witnessed.

The analysis conducted on this device, while promising, is not an entirely accurate estimate of how well it would perform in a real competition. This was proven during the testing, at which time the snowballs had inconsistent launch angles between each throw. The device is extremely dependant on the user maintaining a proper technique that allows the ball to stay in the cup until a specific point in its motion.

The issues brought to light regarding this device may be addressed by making several design modifications. One option to consider is to install a fixed stopper be used to limit the rotation of the throwing arm, instead of relying on an elastic to control the launch point. This would allow for more consistent results in further testing. Additionally, lighter materials may be beneficial to increase the potential speed that the user may activate the arm; this should be considered in the next iterations of the prototype's build.

From the design stages to real-world testing, our group learned about the benefits and drawbacks of certain design decisions that had been made, directly relating to engineering science. By comparing the analysis to the competition trials, we identified possible changes that will be implemented shall this design be developed further.