The Chemistry Maths Book

Erich Steiner

University of Exeter

Second Edition 2008

Solutions

Chapter 1. Numbers, variables, and units

Section 1.2

Calculate and express each result in its simplest form:

1. $3+(-4)=3-4=-1$

2. $3-(-4)=3+4=7$

3. $(-3)-(-4)=-3+4=4-3=1$

4. $(-3)\times(-4)=3\times 4=12$

5. $3\times(-4)=-3\times 4=-12$

6. $8\div(-4)=-8\div 4=-2$

7. $(-8)\div(-4)=8\div 4=2$

8. $\frac{1}{4}+\frac{1}{8}=\frac{2}{8}+\frac{1}{8}=\frac{3}{8}$

9. $\frac{3}{4}-\frac{5}{7}=\frac{3\times 7}{4\times 7}-\frac{4\times 5}{4\times 7}=\frac{21}{28}-\frac{20}{28}=\frac{1}{28}$

10. $\frac{2}{9}-\frac{5}{6}=\frac{4}{18}-\frac{15}{18}=-\frac{11}{18}$

11. $\frac{1}{14}+\frac{2}{21}=\frac{3}{42}+\frac{4}{42}=\frac{7}{42}\xrightarrow[\text{bottom by 7}]{\text{divide top and}}\frac{1}{6}$

12. $\frac{1}{18}-\frac{2}{27}=\frac{3-4}{54}=-\frac{1}{54}$

13. $\frac{11}{12}+\frac{3}{16}=\frac{44+9}{48}=\frac{53}{48}$

14. $\frac{1}{2}\times\frac{3}{4}=\frac{1\times 3}{2\times 4}=\frac{3}{8}$

15. $2\times\frac{3}{4}=\frac{2\times 3}{4}=\frac{6}{4}=\frac{3}{2}$

16. $\frac{2}{3}\times\frac{5}{6}=\frac{10}{18}=\frac{5}{9}$

17. $\left(-\frac{2}{3}\right)\times\left(-\frac{3}{4}\right)=\frac{2}{3}\times\frac{3}{4}=\frac{6}{12}=\frac{1}{2}$

18. $\frac{3}{4}\div\frac{4}{5}=\frac{3}{4}\times\frac{5}{4}=\frac{15}{16}$

19. $\frac{2}{3}\div\frac{5}{3}=\frac{2}{3}\times\frac{3}{5}=\frac{2\times\cancel{3}}{\cancel{3}\times 5}=\frac{2}{5}$

20. $\frac{2}{15} \div \frac{4}{5} = \frac{2 \times 5}{15 \times 4} = \frac{\cancel{10}}{\cancel{10} \times 6} = \frac{1}{6}$

21. $\frac{1}{3} \div \frac{1}{9} = \frac{1}{\cancel{3}} \times \cancel{9}^{3} = 3$

Section 1.3

Factorize in prime numbers:

22. $6 = 2 \times 3$

23. $80 = 16 \times 5 = 2^4 \times 5$

24. $256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$

25. $810 = 10 \times 81 = 2 \times 5 \times 3^4 = 2 \times 3^4 \times 5$

Simplify by factorization and cancellation:

26. $\frac{3}{18} = \frac{\cancel{3}}{\cancel{3} \times 6} = \frac{1}{6}$

27. $\frac{21}{49} = \frac{3 \times \cancel{7}}{7 \times \cancel{7}} = \frac{3}{7}$

28. $\frac{63}{294} = \frac{3 \times \cancel{3} \times \cancel{7}}{2 \times \cancel{3} \times \cancel{7} \times 7} = \frac{3}{14}$

29. $\frac{768}{5120} = \frac{3 \times 2^8}{5 \times 2^{10}} = \frac{3}{5 \times 2^2} = \frac{3}{20}$

Find the value of:

30. $2! = 2 \times 1 = 2$

32. $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 = 5040$

33. $10! = 10 \times 9 \times 8 \times 7! = 720 \times 5040 = 3628800$

Evaluate by cancellation:

33. $\frac{3!}{2!} = \frac{3 \times 2!}{2!} = \frac{3 \times \cancel{2!}}{\cancel{2!}} = 3$

34. $\frac{6!}{3!} = \frac{6 \times 5 \times 4 \times 3!}{3!} = \frac{6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}} = 120$

35. $\dfrac{5!}{3!2!}=\dfrac{5\times4\times3!}{3!\times2}=\dfrac{5\times2\times\cancel{2}\times\cancel{3!}}{\cancel{3!}\times\cancel{2}}=5\times2=10$

36. $\dfrac{10!}{7!3!}=\dfrac{10\times9\times8\times7!}{7!\times3\times2}=\dfrac{10\times3\times\cancel{3}\times4\times\cancel{2}\times\cancel{7!}}{\cancel{7!}\times\cancel{3}\times\cancel{2}}=10\times3\times4=120$

Section 1.4

Express as decimal fractions:

37. $10^{-2}=0.01$

38. $2\times10^{-3}=2\times0.001=0.002$

39. $2+3\times10^{-4}+5\times10^{-6}=2+3\times0.0001+5\times0.000001=2+0.0003+0.000005=2.000305$

40. $\dfrac{3}{8}=3\times\dfrac{1}{8}=3\times0.125=0.375$

41. $\dfrac{1}{25}=\dfrac{4}{100}=0.04$

42. $\dfrac{5}{32}=5\times\dfrac{1}{32}=5\times0.03125=0.15625$

Find the repeating sequence of digits in the nonterminating decimal fraction representation of:

43.

1/9 By long division: → $1/9=0.\boxed{1}\,1\ldots$ ←	$\begin{array}{r} 0.11\ldots \\ 9\,\overline{)\,1.0} \\ \underline{9} \\ 10 \\ 9 \\ \vdots \end{array}$

44.

1/11 By long division: → $1/11=0.\boxed{09}\,09\ldots$ ←	$\begin{array}{r} 0.09\,09\,09\ldots \\ 11\,\overline{)\,1.00} \\ \underline{99} \\ 100 \\ \underline{99} \\ 100 \\ 99 \\ \vdots \end{array}$

45.

$1/21$ By long division: →	$21\overline{)1.00}$ = 0.047619 04…

$$
\begin{array}{r}
0.047619\,04\ldots \\
21\overline{)1.00} \\
\underline{84} \\
160 \\
\underline{147} \\
130 \\
\underline{126} \\
40 \\
\underline{21} \\
190 \\
\underline{189} \\
100 \\
\underline{84} \\
\vdots
\end{array}
$$

$1/21 = 0.\boxed{047619}\,047\ldots$ ←

46.

$1/17$ By long division: →	$17\overline{)1.00}$ = 0.0588235294117647 0588…

$$
\begin{array}{r}
0.0588235294117647\,0588\ldots \\
17\overline{)1.00} \\
\underline{85} \\
150 \\
\underline{136} \\
140 \\
\underline{136} \\
40 \\
\underline{34} \\
60 \\
\underline{51} \\
90 \\
\underline{85} \\
50 \\
\underline{34} \\
160 \\
\underline{153} \\
70 \\
\underline{68} \\
20 \\
\underline{17} \\
30 \\
\underline{17} \\
130 \\
\underline{119} \\
110 \\
\underline{102} \\
80 \\
\underline{68} \\
120 \\
\underline{119} \\
100 \\
\vdots
\end{array}
$$

$1/17 = 0.\boxed{0588235294117647}\,0588\ldots$ ←

Use the rules of rounding to give each of the following to 8, 7, 6, 5, 4, 3, 2 and 1 significant figures:

47. $1/13 = 0.076923076923\ldots$

≈ 0.076923077

≈ 0.07692308

≈ 0.0769231

≈ 0.076923

≈ 0.07692

≈ 0.0769

≈ 0.077

≈ 0.08

48. $\sqrt{2} = 1.414213562373\ldots$

≈ 1.4142136

≈ 1.414214

≈ 1.41421

≈ 1.4142

≈ 1.414

≈ 1.41

≈ 1.4

≈ 1

49. $\pi = 3.141592653589\ldots$

≈ 3.1415927

≈ 3.141593

≈ 3.14159

≈ 3.1416

≈ 3.142

≈ 3.14

≈ 3.1

≈ 3

Section 1.6

Simplify if possible:

50. $a^2a^3 = a^{2+3} = a^5$

51. $a^3a^{-3} = a^{3-3} = a^0 = 1$

52. $a^3a^{-4} = a^{3-4} = a^{-1}$

53. $a^3/a^2 = a^3a^{-2} = a^1 = a$

54. $a^5/a^{-4} = a^5a^4 = a^9$

55. $(a^3)^4 = a^{3\times4} = a^{12}$

56. $(a^2)^{-3} = a^{2\times(-3)} = a^{-6}$

57. $(1/a^2)^{-4} = (a^{-2})^{-4} = a^{(-2)\times(-4)} = a^8$

58. $a^{1/2}a^{1/3} = a^{1/2+1/3} = a^{5/6}$

59. $(a^2)^{3/2} = a^{2\times3/2} = a^3$

60. $(a^3b^6)^{2/3} = a^{3\times2/3}b^{6\times2/3} = a^2b^4$

61. $(a^3+b^3)^{1/3}$

62. $9^{1/2} = \sqrt{9} = 3$

63. $8^{2/3} = (8^2)^{1/3} = 64^{1/3} = 4$ or $8^{2/3} = (8^{1/3})^2 = 2^2 = 4$

64. $32^{3/5} = (32^{1/5})^3 = 2^3 = 8$

65. $27^{-4/3} = (27^{1/3})^{-4} = 3^{-4} = 1/81$

Evaluate:

66. $7-3\times2 = 7-6 = 1$

67. $7-(3\times2) = 7-6 = 1$

68. $(7-3)\times2 = 4\times2 = 8$

69. $7+3\times4-5 = 7+12-5 = 14$

70. $(7+3)\times4-5 = 10\times4-5 = 40-5 = 35$

71. $4\div2\times7-2 = 2\times7-2 = 14-2 = 12$

72. $4\div2+7\times2 = 2+14 = 16$

73. $8\times2\div4\div2 = 16\div4\div2 = 4\div2 = 2$

74. $3+4^2 = 3+16 = 19$

75. $3+4\times5^2 = 3+4\times25 = 3+100 = 103$

76. $25+144^{1/2} = 25+12 = 37$

77. $(5^2+12^2)^{1/2} = (25+144)^{1/2} = 169^{1/2} = 13$

Section 1.7

Find the sum and product of the pairs of complex numbers:

78. $z_1 = 3+5i,\ z_2 = 4-7i$, $\quad z_1+z_2 = 3+5i+4-7i = (3+4)+(5-7)i = 7-2i$, 125

79. $z_1 = 1-6i,\ z_2 = -5-4i$, $\quad z_1+z_2 = 1-6i-5-4i = (1-5)+(-6-4)i = -4-10i$,

$$z_1z_2 = (1-6i)(-5-4i) = -5-4i+30i+24i^2 = -5-4i+30i-24 = (-5-24)+(-4+30)i = -29+26i$$

Section 1.8 Units

For each of the following dimensions give its SI unit in terms of base units (column 5 of Table 1.1) and, where possible, in terms of the derived units in Table 1.2; identify a physical quantity for each:

80. L^3 : m^3, volume

81. ML^{-3} : $\mathrm{kg\ m^{-3}}$, mass per unit volume = density

82. NL^{-3} : $\mathrm{mol\ m^{-3}}$, amount of substance per unit volume = concentration

83. MLT^{-1} : $\mathrm{kg\ m\ s^{-1}}$, mass × velocity = momentum

84. MLT^{-2} : $\mathrm{kg\ m\ s^{-2}} = \mathrm{N}$, mass × acceleration = force

85. $\mathsf{ML}^2\mathsf{T}^{-2}$: $\mathrm{kg\ m^2 s^{-2}} = \mathrm{N\ m} = \mathrm{J}$, force × distance = work, energy

86. $\mathsf{ML}^{-1}\mathsf{T}^{-2}$: $\mathrm{kg\ m^{-1}s^{-2}} = \mathrm{kg\ m\ s^{-2}}/\mathrm{m^2} = \mathrm{N}/\mathrm{m^2} = \mathrm{Pa}$, force per unit area = pressure

87. IT : A s = C, electric current × time = electric charge

88. $\mathsf{ML}^2\mathsf{I}^{-1}\mathsf{T}^{-3}$: $\mathrm{kg\ m^2A^{-1}s^{-3}} = \mathrm{J\ C^{-1}} = \mathrm{V}$, work per unit charge = electric potential

89. $\mathsf{ML}^2\mathsf{T}^{-2}\mathsf{N}^{-1}$: $\mathrm{kg\ m^2s^{-2}mol^{-1}} = \mathrm{J\ mol^{-1}}$, energy per mole = molar energy

90. $\mathsf{ML}^2\mathsf{T}^{-2}\mathsf{N}^{-1}\theta^{-1}$: $\mathrm{kg\ m^2s^{-2}mol^{-1}K^{-1}} = \mathrm{J\ mol^{-1}K^{-1}}$,
molar energy per unit temperature = heat capacity, molar entropy

91. Given that 1 mile (mi) is 1760 yd and 1 hour (h) is 60 min, express a speed of 60 miles per hour in **(i)** m s^{-1}, **(ii)** km h^{-1}.

(i) We have $60\ \text{mi h}^{-1} = 60\times(1760\ \text{yd})\times(60\ \text{min})^{-1} = 1760\ \text{yd min}^{-1}$.

Now $1\ \text{yd} = 0.9144\ \text{m}$ and $1\ \text{min} = 60\ \text{s}$.

Therefore $1\ \text{yd min}^{-1} = (0.9144\ \text{m})\times(60\ \text{s})^{-1} = 0.01524\ \text{m s}^{-1}$

and $60\ \text{mi h}^{-1} = 1760\times 0.01524\ \text{m s}^{-1} = 26.8224\ \text{m s}^{-1}$

(ii) We have
$$26.8224\ \text{m s}^{-1} = (26.8224\times10^{-3}\ \text{km})\times\left(\frac{1}{3600}\ \text{h}\right)^{-1}$$
$$= 26.8224\times10^{-3}\times3600\ \text{km h}^{-1} = 96.5606\ \text{h}^{-1}$$

Therefore $60\ \text{mi h}^{-1} = 96.5606\ \text{km h}^{-1}$

92. **(i)** What is the unit of velocity in a system in which the unit of length is the inch $(1\ \text{in} = 2.54\times10^{-2}\ \text{m})$ and the unit of time is the hour (h)? **(ii)** Express this in terms of base SI units. **(iii)** A snail travels at speed $1.2\ \text{in min}^{-1}$. Express this in units yd h^{-1}, m s^{-1}, and km h^{-1}.

(i) in h^{-1}

(ii) $\text{in h}^{-1} = (2.54\times10^{-2}\ \text{m})\times(3600\ \text{s})^{-1} = 7.0556\times10^{-6}\ \text{m s}^{-1}$

(iii) We have $1\ \text{yd} = 0.9144\ \text{m}$ and $1\ \text{min} = 60\ \text{s}$.

Therefore
$$1.2\ \text{in min}^{-1} = 1.2\times\left(\frac{1}{36}\ \text{yd}\right)\times\left(\frac{1}{60}\ \text{h}\right)^{-1} = \frac{1.2\times60}{36}\text{yd h}^{-1}$$
$$= 2\ \text{yd h}^{-1}$$

$$2\ \text{yd h}^{-1} = 2\times(0.9144\ \text{m})\times(3600\ \text{s})^{-1}$$
$$= 5.08\times10^{-4}\ \text{m s}^{-1}$$

$$5.08\times10^{-4}\ \text{m s}^{-1} = 5.08\times10^{-4}\times(10^{-3}\ \text{km})\times\left(\frac{1}{3600}\ \text{h}\right)^{-1}$$
$$= 1.8288\times10^{-3}\ \text{km h}^{-1}$$

93. The non-SI unit of mass called the (international avoirdupois) pound has value $1\text{ lb} = 0.45359237\text{ kg}$. The "weight" of the mass in the presence of gravity is called the pound-force, lbf. Assuming that the acceleration of gravity is $g = 9.80665\text{ m s}^{-2}$, **(i)** express lbf in SI units, **(ii)** express, in SI units, the pressure that is denoted (in some parts of the world) by $\text{psi} = \text{lbf in}^{-2}$, **(iii)** calculate the work done (in SI units) in moving a body of mass 200 lb through distance 5 yd against the force of gravity.

(i) Force is $\text{mass} \times \text{acceleration}$.

Therefore
$$\begin{aligned} 1\text{ lbf} &= 1\text{ lb} \times g = (0.45359237\text{ kg}) \times (9.80665\text{ m s}^{-2}) \\ &= 4.448222\text{ N} \end{aligned}$$

(ii)
$$\begin{aligned} 1\text{ psi} &= 1\text{ lbf in}^{-2} = (4.448222\text{ N}) \big/ (2.54 \times 10^{-2}\text{ m})^2 \\ &= 6894.75729\text{ Pa} \end{aligned}$$

(iii) Work is $\text{force} \times \text{distance}$.

Therefore
$$\begin{aligned} \text{work done} &= (200\text{ lbf}) \times (5 \times 36\text{ in}) \\ &= (200 \times 4.448222\text{ N}) \times (5 \times 36 \times 2.54 \times 10^{-2}\text{ m}) = 4067.454\text{ J} \\ &= 4.067454\text{ kJ} \end{aligned}$$

94. The vapour pressure of water at 20°C is recorded as $p(H_2O, 20^\circ C) = 17.5\text{ Torr}$. Express this in terms of **(i)** the base SI unit of pressure, **(ii)** bar, **(iii)** atm.

(i) We have $\text{Torr} = 133.322\text{ Pa}$

Therefore
$$\begin{aligned} 17.5\text{ Torr} &= 17.5 \times 133.322\text{ Pa} \\ &= 2333.1\text{ Pa} = 2.3331\text{ kPa} \end{aligned}$$

(ii) We have $\text{bar} = 10^5\text{ Pa}$

Therefore
$$\begin{aligned} 17.5\text{ Torr} &= 2333.1 \times 10^{-5}\text{ bar} \\ &= 2.3331 \times 10^{-2}\text{ bar} \end{aligned}$$

(iii) We have $\text{atm} = 101325\text{ Pa} = 1.01325\text{ bar}$

Therefore
$$\begin{aligned} 17.5\text{ Torr} &= \frac{2.3331 \times 10^{-2}}{1.01325}\text{ atm} \\ &= 2.3026 \times 10^{-2}\text{ atm} \end{aligned}$$

95. The root mean square speed of the particles of an ideal gas at temperature T is $c = (3RT/M)^{1/2}$, where $R = 8.31447\ \text{J K}^{-1}\ \text{mol}^{-1}$ and M is the molar mass. Confirm that c has dimensions of velocity.

quantity	type	dimensions
R	$\dfrac{\text{energy}}{\text{temperature} \times \text{amount of substance}}$	$[\mathsf{ML^2T^{-2}}]\times[\theta^{-1}]\times[\mathsf{N^{-1}}]$
T	temperature	θ
M	molar mass	$\mathsf{MN^{-1}}$
$(RT/M)^{1/2}$	velocity ←	$\left[\dfrac{[\mathsf{ML^2T^{-2}}\,\theta^{-1}\,\mathsf{N^{-1}}]\times[\theta]}{\mathsf{M\,N^{-1}}}\right]^{1/2} = \left[\mathsf{L^2T^{-2}}\right]^{1/2}$ $= \mathsf{LT^{-1}}$

Express in base SI units

96. $\text{dm}^{-3} = (10^{-1}\ \text{m})^{-3} = (10^{-1})^{-3} \times \text{m}^{-3}$
$= 10^{3}\ \text{m}^{-3}$

97. $\text{cm ms}^{-2} = (10^{-2}\,\text{m}) \times (10^{-3}\ \text{s})^{-2} = 10^{-2}\,\text{m} \times 10^{6}\ \text{s}^{-2}$
$= 10^{4}\ \text{m s}^{-2}$

98. $\text{g dm}^{-3} = (10^{-3}\ \text{kg}) \times (10^{-1}\ \text{m})^{-3} = 10^{-3} \times 10^{3}\ \text{kg m}^{-3}$
$= \text{kg m}^{-3}$

99. $\text{mg pm}\ \mu\text{s}^{-2} = (10^{-3} \times 10^{-3}\ \text{kg}) \times (10^{-12}\ \text{m}) \times (10^{-6}\,\text{s})^{-2}$
$= 10^{-6}\ \text{kg m s}^{-2} = 10^{-6}\ \text{N}$

100. $\text{dg mm}^{-1}\ \text{ns}^{-2} = (10^{-1} \times 10^{-3}\ \text{kg}) \times (10^{-3}\ \text{m})^{-1} \times (10^{-9}\ \text{s})^{-2} = \left(\dfrac{10^{-4}}{10^{-3} \times 10^{-18}}\right)\ \text{kg m}^{-1}\text{s}^{-2}$
$= 10^{17}\ \text{kg m}^{-1}\text{s}^{-2} = 10^{17}\ \text{Pa}$

101. $\text{GHz}\ \mu\text{m} = (10^{9}\ \text{s}^{-1}) \times (10^{-6}\ \text{m})$
$= 10^{3}\ \text{m s}^{-1} = 1\ \text{km s}^{-1}$

102. $\text{kN dm} = (10^{3}\ \text{kg m s}^{-2}) \times (10^{-1}\ \text{m})$
$= 10^{2}\ \text{kg m}^{2}\ \text{s}^{-2} = 10^{2}\ \text{J}$

103. $\text{mmol dm}^{-3} = (10^{-3}\ \text{mol}) \times (10^{-1}\ \text{m})^{-3} = (10^{-3}\ \text{mol}) \times (10^{3}\ \text{m}^{-3})$
$= \text{mol m}^{-3}$

104. Given relative atomic masses $A_r(^{14}\text{N}) = 14.0031$ and $A_r(^{1}\text{H}) = 1.0078$, calculate **(i)** the relative molar mass of ammonia, $M_r(^{14}\text{N}^{1}\text{H}_3)$, **(ii)** the molecular mass and **(iii)** the molar mass.

(i) $M_r(^{14}\text{N}^{1}\text{H}_3) = A_r(^{14}\text{N}) + 3\times A_r(^{1}\text{H})$
$= 14.0031 + 3\times 1.0078 = 17.0265$

(ii) $m(^{14}\text{N}^{1}\text{H}_3) = 17.0265\text{ u} = 17.0265\times 1.66054\times 10^{-27}\text{ kg}$
$= 28.2732\times 10^{-27}\text{ kg}$

(iii) $M(^{14}\text{N}^{1}\text{H}_3) = 17.0265\text{ g mol}^{-1} = 0.01703\text{ kg mol}^{-1}$

105. The bond length of HCl is $R_e = 1.2745\times 10^{-10}\text{ m}$ and the relative atomic masses are $A_r(^{35}\text{Cl}) = 34.9688$ and $A_r(^{1}\text{H}) = 1.0078$. **(i)** Express the bond length in (a) pm, (b) Å and (c) a_0 . Calculate **(ii)** the reduced mass of the molecule and **(iii)** its moment of inertia.

(i) (a) $\text{pm} = 10^{-12}\text{ m},\ \text{m} = 10^{12}\text{ pm} \quad\rightarrow\quad R_e = 1.2745\times 10^{-10}\text{ m} = 1.2745\times 10^{-10}\times 10^{12}\text{ pm} = 127.45\text{ pm}$

(b) $\text{Å} = 10^{-10}\text{ m},\ \text{m} = 10^{10}\text{ Å} \quad\rightarrow\quad R_e = 1.2745\times 10^{-10}\text{ m} = 1.2745\text{ Å}$

(c) $a_0 = 5.29177\times 10^{-11}\text{ m} \quad\rightarrow\quad R_e = \dfrac{1.2745\times 10^{-10}}{5.29177\times 10^{-11}}\,a_0 = 2.4808\,a_0$

(ii) The reduced mass of $^{1}\text{H}^{35}\text{Cl}$ is

$$\mu = \frac{m(^{1}\text{H})\times m(^{35}\text{Cl})}{m(^{1}\text{H}) + m(^{35}\text{Cl})}$$

where $m(^{1}\text{H}) = A_r(^{1}\text{H})\times\text{u} = 1.0078\text{ u}$ and $m(^{35}\text{Cl}) = A_r(^{35}\text{Cl})\times\text{u} = 34.9688\text{ u}$ are the molecular masses and $\text{u} = 1.66054\times 10^{-27}\text{ kg}$ is the unified atomic mass unit. Then

$$\mu = \frac{1.0078\times 34.9688}{1.0078 + 34.9688}\times\frac{\text{u}^{\not 2}}{\not\text{u}} = 0.97957\times 1.66054\times 10^{-27}\text{ kg}$$
$$= 1.6266\times 10^{-27}\text{ kg}$$

(iii) The moment of inertia of the molecule is

$$I = \mu R_e^2 = (1.6266\times 10^{-27}\text{ kg})\times(1.2745\times 10^{-10}\text{ m})^2$$
$$= 2.6421\times 10^{-47}\text{ kg m}^2$$

106. The origin of the fundamental aborption band in the vibration-rotation spectrum of $^{1}H^{35}Cl$ lies at wavenumber $\tilde{\nu} = 2886\ \text{cm}^{-1}$. Calculate the corresponding **(i)** frequency, **(ii)** wavelength, and **(iii)** energy in units of eV and kJ mol^{-1}.

We have $\tilde{\nu} = \dfrac{1}{\lambda} = \dfrac{\nu}{c}$ where λ is the wavelength, ν is the frequency, and *c* is the speed of light. Then

(i) $\nu = c \times \tilde{\nu} = (2.99792 \times 10^{10}\ \text{cm s}^{-1}) \times (2886\ \text{cm}^{-1})$
$= 8.652 \times 10^{13}\ \text{s}^{-1} = 86.52\ \text{THz}$

(ii) $\lambda = \dfrac{\text{cm}}{2886} = 3.465 \times 10^{-4}\ \text{cm} = 3.465 \times 10^{-6}\ \text{m} = 3.465\ \mu\text{m}$

(iii) We have $\text{eV} = 1.60218 \times 10^{-19}\ \text{J} \equiv 96.486\ \text{kJ mol}^{-1} \equiv 8065.5\ \text{cm}^{-1}$.

Therefore $\text{cm}^{-1} \equiv \dfrac{1}{8065.5}\text{eV} \equiv \dfrac{96.486}{8065.5}\ \text{kJ mol}^{-1}$

and $2886\ \text{cm}^{-1} \equiv \dfrac{2886}{8065.5}\ \text{eV} = 0.3578\ \text{eV}$

$$\equiv \frac{2886 \times 96.486}{8065.5}\ \text{kJ mol}^{-1} = 34.52\ \text{kJ mol}^{-1}$$

107. In the kinetic theory of gases, the mean speed of the particles of gas at temperature *T* is $\bar{c} = \left(8RT/\pi M\right)^{1/2}$, where *M* is the molar mass. **(i)** Perform an order-of-magnitude calculation of $\bar{c}$ for N_2 at 298.15 K ($M = 28.01\ \text{g mol}^{-1}$). **(ii)** Calculate $\bar{c}$ to 3 significant figures.

We have $R = 8.31447\ \text{J K}^{-1}\ \text{mol}^{-1}$, $T = 298.15\ \text{K}$, $M = 28.01\ \text{g mol}^{-1}$.

(i) Let $R \approx 8\ \text{J K}^{-1}\ \text{mol}^{-1}$, $T \approx 300$ K, $\pi \approx 3$, and $M \approx 30 \times 10^{-3}\ \text{kg mol}^{-1}$. Then

$$\bar{c} = \left(\frac{8RT}{\pi M}\right)^{1/2} \approx \left(\frac{8 \times 8 \times 300}{3 \times 30 \times 10^{-3}}\right)^{1/2} \times \left(\frac{\text{J K}^{-1}\ \text{mol}^{-1} \times \text{K}}{\text{kg mol}^{-1}}\right)^{1/2}$$
$$\approx 462\ \text{m s}^{-1}$$

(ii) $$\bar{c} = \left(\frac{8RT}{\pi M}\right)^{1/2} = \left(\frac{8 \times 8.31447 \times 298.15}{\pi \times 28.01 \times 10^{-3}}\right)^{1/2}\ \text{m s}^{-1}$$
$$= 475\ \text{m s}^{-1}$$

108. In the Bohr model of the ground state of the hydrogen atom, the electron moves round the nucleus in a circular orbit of radius $a_0 = 4\pi\varepsilon_0\hbar^2 / m_e e^2$, now called the Bohr (radius). Given $\varepsilon_0 = 8.85419 \times 10^{-12}\ \text{F m}^{-1}$, use the units and values of m_e, e and $\hbar$ given in Table 1.4 to confirm **(i)** that a_0 is a length, and **(ii)** the value of a_0 in Table 1.4.

From Table 4, $\hbar = 1.05457 \times 10^{-34}\ \text{J s}$

$m_e = 9.10938 \times 10^{-31}\ \text{kg}$

$e = 1.60218 \times 10^{-19}\ \text{C}$

(i) The unit of a_0 is

$$\frac{(\text{F m}^{-1}) \times (\text{J s})^2}{(\text{kg}) \times (\text{C})^2} = \text{F m}^{-1}\ \text{J}^2\ \text{s}^2\ \text{kg}^{-1}\ \text{C}^{-2}.$$

From Table 1.2 ,

$$\text{J} = \text{kg m}^2\text{s}^{-2},\ \text{F} = \text{CV}^{-1} = \text{C}^2\text{J}^{-1}$$

Therefore

$$\begin{aligned}\text{F m}^{-1}\ \text{J}^2\ \text{s}^2\ \text{kg}^{-1}\ \text{C}^{-2} &= (\cancel{\text{C}^2}\ \cancel{\text{J}^{-1}})\ \text{m}^{-1}\ \text{J}^{\cancel{2}}\ \text{s}^2\ \text{kg}^{-1}\ \cancel{\text{C}^{-2}} \\ &= \text{m}^{-1}\ \text{J s}^2\ \text{kg}^{-1} \\ &= \cancel{\text{m}^{-1}}\ (\cancel{\text{kg}}\ \text{m}^{\cancel{2}}\ \cancel{\text{s}^{-2}})\ \cancel{\text{s}^2}\ \cancel{\text{kg}^{-1}} \\ &= \text{m}\end{aligned}$$

and a_0 is a length.

(ii) $$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = \frac{4 \times 3.14159 \times 8.85419 \times 10^{-12} \times (1.05457 \times 10^{-34})^2}{9.10938 \times 10^{-31} \times (1.60218 \times 10^{-19})^2}\ \text{m}$$

$$= \frac{123.7397 \times 10^{-80}}{23.38360 \times 10^{-69}} = 5.29173 \times 10^{-11}\ \text{m}$$

The small difference between this and the tabulated value comes from rounding errors (see Sections 1.4 and 20.2).