Chapter 13

Introduction to nonstationary time series

13.1 Overview

This chapter begins by defining the concepts of stationarity and nonstationarity as applied to univariate time series and, in the case of nonstationary series, the concepts of difference-stationarity and trend-stationarity. It next describes the consequences of nonstationarity for models fitted using nonstationary time-series data and gives an account of the Granger–Newbold Monte Carlo experiment with random walks. Next the two main methods of detecting nonstationarity in time series are described, the graphical approach using correlograms and the more formal approach using Augmented Dickey–Fuller unit root tests. This leads to the topic of cointegration. The chapter concludes with a discussion of methods for fitting models using nonstationary time series: detrending, differencing, and error-correction models.

13.2 Learning outcomes

After working through the corresponding chapter in the text, studying the corresponding slideshows, and doing the starred exercises in the text and the additional exercises in this subject guide, you should be able to:

- explain what is meant by stationarity and nonstationarity.
- explain what is meant by a random walk and a random walk with drift
- derive the condition for the stationarity of an AR(1) process
- explain what is meant by an integrated process and its order of integration
- explain why Granger and Newbold obtained the results that they did
- explain what is depicted by a correlogram
- perform an Augmented Dickey–Fuller unit root test to test a time series for nonstationarity
- test whether a set of time series are cointegrated
- construct an error-correction model and describe its advantages over detrending and differencing.

13.3 Further material

Addition to Section 13.6 Cointegration

Section 13.6 contains the following paragraph on page 507:

In the case of a cointegrating relationship, least squares estimators can be shown to be superconsistent (Stock, 1987). An important consequence is that OLS may be used to fit a cointegrating relationship, even if it belongs to a system of simultaneous relationships, for any simultaneous equations bias tends to zero asymptotically.

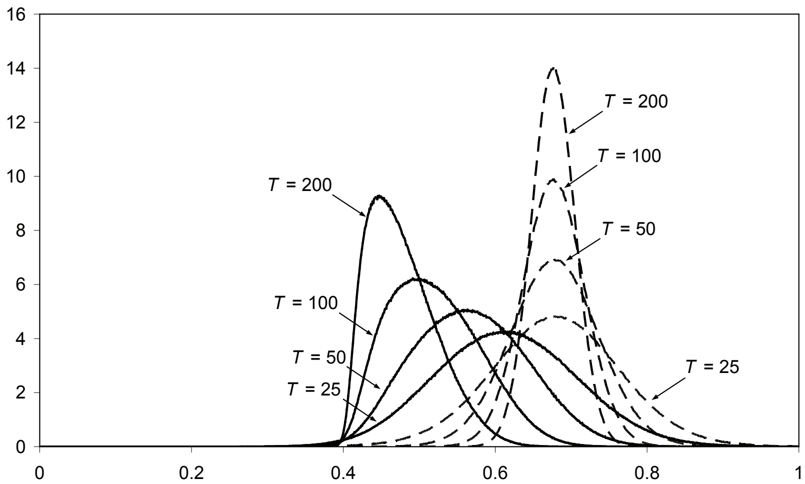
This cries out for an illustrative simulation, so here is one. Consider the model:

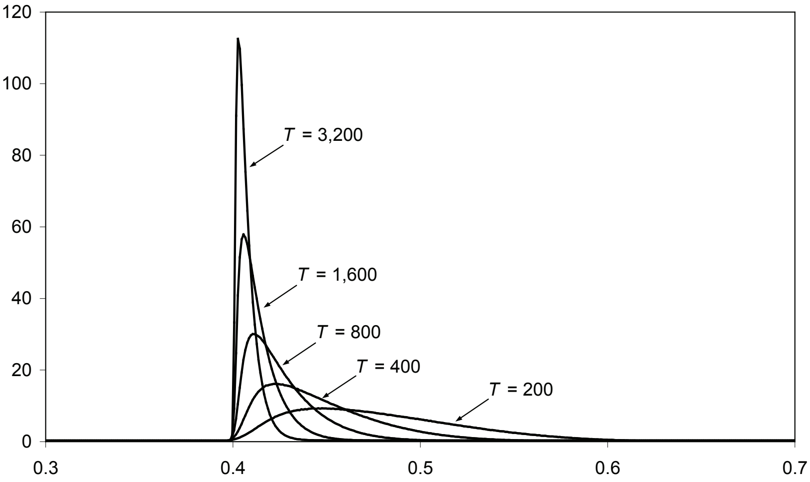
$$\begin{aligned} Y_t &= \beta_1 + \beta_2 X_t + \beta_3 Z_t + \varepsilon_{Yt} \\ X_t &= \alpha_1 + \alpha_2 Y_t + \varepsilon_{Xt} \\ Z_t &= \rho Z_{t-1} + \varepsilon_{Zt} \end{aligned}$$

where Y_t and X_t are endogenous variables, Z_t is exogenous, and ε_{Yt}, ε_{Xt}, and ε_{Zt} are iid $N(0,\, 1)$ disturbance terms. We expect OLS estimators to be inconsistent if used to fit either of the first two equations. However, if $\rho = 1$, Z is nonstationary, and X and Y will also be nonstationary. So, if we fit the second equation, for example, the OLS estimator of α_2 will be superconsistent. This is illustrated by a simulation where the first two equations are:

$$\begin{aligned} Y_t &= 1.0 + 0.8 X_t + 0.5 Z_t + \varepsilon_{Yt} \\ X_t &= 2.0 + 0.4 Y_t + \varepsilon_{Xt}. \end{aligned}$$

The distributions in the right of the figure below (dashed lines) are for the case $\rho = 0.5$. Z is stationary, and so are Y and X. You will have no difficulty in demonstrating that plim $\widehat{\alpha}_2^{OLS} = 0.68$. The distributions to the left of the figure (solid lines) are for $\rho = 1$, and you can see that in this case the estimator is consistent. But is it superconsistent? The variance seems to be decreasing relatively slowly, not fast, especially for small sample sizes. The explanation is that the superconsistency becomes apparent only for very large sample sizes, as shown in the second figure.





13.4 Additional exercises

A13.1 The Figure 13.1 plots the logarithm of the US population for the period 1959–2003. It is obviously nonstationary. Discuss whether it is more likely to be difference-stationary or trend-stationary.

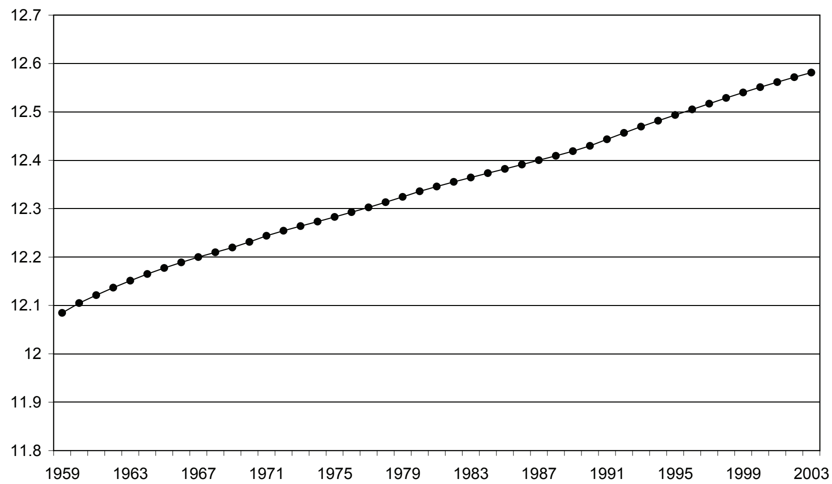


Figure 13.1: Logarithm of the US population.

A13.2 Figure 13.2 plots the first difference of the logarithm of the US population for the period 1959–2003. Explain why the vertical axis measures the proportional growth rate. Comment on whether the series appears to be stationary or nonstationary.

A13.3 The regression output below shows the results of ADF unit root tests on the logarithm of the US population, and its difference, for the period 1959–2003. Comment on the results and state whether they confirm or contradict your conclusions in Exercise A13.2.

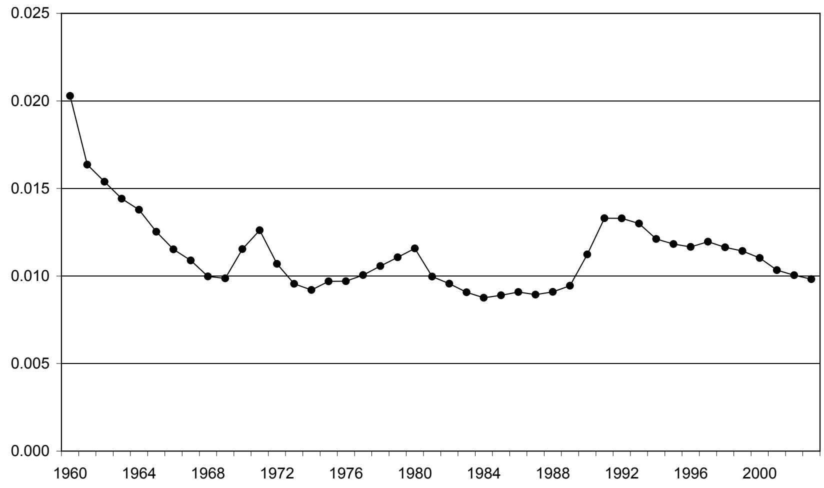


Figure 13.2: Logarithm of the US population, first difference.

```
      Augmented Dickey--Fuller Unit Root Test on LGPOP
============================================================
Null Hypothesis: LGPOP has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Fixed)
============================================================
                                        t-Statistic  Prob.*
============================================================
Augmented Dickey--Fuller test statistic   -2.030967   0.5682
Test critical values1% level             -4.186481
                   5% level              -3.518090
                  10% level              -3.189732
============================================================
*MacKinnon (1996) one-sided p-values.

Augmented Dickey--Fuller Test Equation
Dependent Variable: D(LGPOP)
Method: Least Squares
Sample(adjusted): 1961 2003
Included observations: 43 after adjusting endpoints
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
     LGPOP(-1)      -0.047182   0.023231  -2.030967   0.0491
   D(LGPOP(-1))      0.687772   0.058979   11.66139   0.0000
         C           0.574028   0.281358   2.040209   0.0481
   @TREND(1959)      0.000507   0.000246   2.060295   0.0461
============================================================
R-squared            0.839263    Mean dependent var 0.011080
Adjusted R-squared   0.826898    S.D. dependent var 0.001804
S.E. of regression   0.000750    Akaike info criter-11.46327
Sum squared resid    2.20E-05    Schwarz criterion -11.29944
Log likelihood       250.4603    F-statistic        67.87724
Durbin-Watson stat   1.164933    Prob(F-statistic)  0.000000
============================================================
```

```
      Augmented Dickey--Fuller Unit Root Test on DLGPOP
============================================================
Null Hypothesis: DLGPOP has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Fixed)
============================================================
                                        t-Statistic  Prob.*
============================================================
Augmented Dickey--Fuller test statistic   -2.513668   0.3203
Test critical values1% level             -4.192337
                   5% level             -3.520787
                  10% level             -3.191277
============================================================
*MacKinnon (1996) one-sided p-values.

Augmented Dickey--Fuller Test Equation
Dependent Variable: D(DLGPOP)
Method: Least Squares
Sample(adjusted): 1962 2003
Included observations: 42 after adjusting endpoints
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
    DLGPOP(-1)      -0.161563   0.064274  -2.513668   0.0163
   D(DLGPOP(-1))     0.294717   0.117766   2.502573   0.0167
         C           0.001714   0.000796   2.152327   0.0378
   @TREND(1959)     -1.32E-07   9.72E-06  -0.013543   0.9893
============================================================
R-squared             0.320511    Mean dependent var-0.000156
Adjusted R-squared    0.266867    S.D. dependent var 0.000827
S.E. of regression    0.000708    Akaike info criter-11.57806
Sum squared resid     1.90E-05    Schwarz criterion -11.41257
Log likelihood        247.1393    F-statistic        5.974780
Durbin-Watson stat    1.574084    Prob(F-statistic)  0.001932
============================================================
```

A13.4 A researcher believes that a time series is generated by the process:

$$X_t = \rho X_{t-1} + \varepsilon_t$$

where ε_t is a white noise series generated randomly from a normal distribution with mean zero, constant variance, and no autocorrelation. Explain why the null hypothesis for a test of nonstationarity is that the series is nonstationary, rather than stationary.

A13.5 A researcher correctly believes that a time series is generated by the process:

$$X_t = \rho X_{t-1} + \varepsilon_t$$

where ε_t is a white noise series generated randomly from a normal distribution with mean zero, constant variance, and no autocorrelation. Unknown to the researcher, the true value of ρ is 0.7. The researcher uses a unit root test to test the series for nonstationarity. The output is shown. Discuss the result of the test.

```
        Augmented Dickey--Fuller Unit Root Test on X
============================================================
ADF Test Statistic -2.528841     1%   Critical Value*-3.6289
                                 5%   Critical Value -2.9472
                                10%   Critical Value -2.6118
============================================================
*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey--Fuller Test Equation
Dependent Variable: D(X)
Method: Least Squares
Sample(adjusted): 2 36
Included observations: 35 after adjusting endpoints
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
       X(-1)        -0.379661   0.150132  -2.528841   0.0164
         C           0.222066   0.203435   1.091580   0.2829
============================================================


R-squared             0.162331    Mean dependent var-0.052372
Adjusted R-squared    0.136947    S.D. dependent var 1.095782
S.E. of regression    1.017988    Akaike info criteri2.928979
Sum squared resid     34.19792    Schwarz criterion  3.017856
Log likelihood       -49.25714    F-statistic        6.395035
Durbin-Watson stat    1.965388    Prob(F-statistic)  0.016406
============================================================
```

A13.6 Test of cointegration. Perform a logarithmic regression of expenditure on your commodity on income, relative price, and population. Save the residuals and test them for stationarity. (Note: the critical values in the regression output do not apply to tests of cointegration. For the correct critical values, see the text.)

A13.7 A variable Y_t is generated by the autoregressive process:

$$Y_t = \beta_1 + \beta_2 Y_{t-1} + \varepsilon_t$$

where $\beta_2 = 1$ and ε_t satisfies the regression model assumptions. A second variable Z_t is generated as the lagged value of Y_t:

$$Z_t = Y_{t-1}.$$

Show that Y and Z are nonstationary processes. Show that nevertheless they are cointegrated.

A13.8 X_t and Z_t are independent I(1) (integrated of order 1) time series. W_t is a stationary time series. Y_t is generated as the sum of X_t, Z_t, and W_t. Not knowing this, a researcher regresses Y_t on X_t and Z_t. Explain whether he would find a cointegrating relationship.

A13.9 Two random walks RA_t and RB_t, and two stationary processes SA_t and SB_t are generated by the following processes:

$$\begin{aligned} RA_t &= RA_{t-1} + \varepsilon_{1t} \\ RB_t &= RB_{t-1} + \varepsilon_{2t} \\ SA_t &= \rho_A SA_{t-1} + \varepsilon_{3t}, \quad 0 < \rho_A < 1 \\ SB_t &= \rho_B SB_{t-1} + \varepsilon_{4t}, \quad 0 < \rho_B < 1 \end{aligned}$$

where ε_{1t}, ε_{2t}, ε_{3t}, and ε_{4t}, are iid $N(0, 1)$ (independently and identically distributed from a normal distribution with mean 0 and variance 1).

- Two series XA_t and XB_t are generated as:

$$\begin{aligned} XA_t &= RA_t + SA_t \\ XB_t &= RB_t + SB_t. \end{aligned}$$

Explain whether it is possible for XA_t and XB_t to be stationary.

Explain whether it is possible for them to be cointegrated.

- Two series YA_t and YB_t are generated as:

$$\begin{aligned} YA_t &= RA_t + SA_t \\ YB_t &= RA_t + SB_t. \end{aligned}$$

Explain whether it is possible for YA_t and YB_t to be cointegrated.

- Two series ZA_t and ZB_t are generated as:

$$\begin{aligned} ZA_t &= RA_t + RB_t + SA_t \\ ZB_t &= RA_t - RB_t + SB_t. \end{aligned}$$

Explain whether it is possible for ZA_t and ZB_t to be stationary.

Explain whether it is possible for them to be cointegrated.

13.5 Answers to the starred exercises in the textbook

13.1 Demonstrate that the MA(1) process:

$$X_t = \varepsilon_t + \alpha_2 \varepsilon_{t-1}$$

is stationary. Does the result generalise to higher-order MA processes?

Answer:

The expected value of X_t is zero and therefore independent of time:

$$E(X_t) = E(\varepsilon_t + \alpha_2 \varepsilon_{t-1}) = E(\varepsilon_t) + \alpha_2 E(\varepsilon_{t-1}) = 0 + 0 = 0.$$

Since ε_t and ε_{t-1} are uncorrelated:

$$\sigma^2_{X_t} = \sigma^2_{\varepsilon_t} + \alpha_2^2 \sigma^2_{\varepsilon_{t-1}}$$

and this is independent of time. Finally, because:

$$X_{t-1} = \varepsilon_{t-1} + \alpha_2 \varepsilon_{t-2},$$

the population covariance of X_t and X_{t-1} is given by:

$$\sigma_{X_t, X_{t-1}} = \alpha_2 \sigma^2_\varepsilon.$$

This is fixed and independent of time. The population covariance between X_t and X_{t-s} is zero for all $s > 1$ since then X_t and X_{t-1} have no elements in common. Thus the third condition for stationarity is also satisfied.

All MA processes are stationary, the general proof being a simple extension of that for the MA(1) case.

13.2 A stationary AR(1) process:

$$X_t = \beta_1 + \beta_2 X_{t-1} + \varepsilon_t$$

with $|\beta_2| < 1$, has initial value X_0, where X_0 is defined as:

$$X_0 = \frac{\beta_1}{1-\beta_2} + \sqrt{\frac{1}{1-\beta_2^2}}\varepsilon_0.$$

Demonstrate that X_0 is a random draw from the ensemble distribution for X.

Answer:

Lagging and substituting, it was shown, equation (13.12), that:

$$X_t = \beta_2^t X_0 + \beta_1 \frac{1-\beta_2^t}{1-\beta_2} + \beta_2^{t-1}\varepsilon_1 + \cdots + \beta_2^2 \varepsilon_{t-2} + \beta_2 \varepsilon_{t-1} + \varepsilon_t.$$

With the stochastic definition of X_0, we now have:

$$\begin{aligned} X_t &= \beta_2^t \left(\frac{\beta_1}{1-\beta_2} + \sqrt{\frac{1}{1-\beta_2^2}}\varepsilon_0 \right) + \beta_1 \frac{1-\beta_2^t}{1-\beta_2} + \beta_2^{t-1}\varepsilon_1 + \cdots + \beta_2^2 \varepsilon_{t-2} + \beta_2 \varepsilon_{t-1} + \varepsilon_t \\ &= \frac{\beta_1}{1-\beta_2} + \beta_2^t \sqrt{\frac{1}{1-\beta_2^2}}\varepsilon_0 + \beta_2^{t-1}\varepsilon_1 + \cdots + \beta_2^2 \varepsilon_{t-2} + \beta_2 \varepsilon_{t-1} + \varepsilon_t. \end{aligned}$$

Hence:

$$E(X_t) = \frac{\beta_1}{1-\beta_2}$$

and:

$$\begin{aligned} \text{var}(X_t) &= \text{var}\left(\beta_2^t \sqrt{\frac{1}{1-\beta_2^2}}\varepsilon_0 + \beta_2^{t-1}\varepsilon_1 + \cdots + \beta_2^2 \varepsilon_{t-2} + \beta_2 \varepsilon_{t-1} + \varepsilon_t \right) \\ &= \frac{\beta_2^{2t}}{1-\beta_2^2}\sigma^2_\varepsilon + \left(\beta_2^{2t-2} + \cdots + \beta_2^4 + \beta_2^2 + 1 \right) \sigma^2_\varepsilon \\ &= \frac{\beta_2^{2t}}{1-\beta_2^2}\sigma^2_\varepsilon + \frac{1-\beta_2^{2t}}{1-\beta_2^2}\sigma^2_\varepsilon = \frac{\sigma^2_\varepsilon}{1-\beta_2^2}. \end{aligned}$$

Given the generating process for X_0, one has:

$$E(X_0) = \frac{\beta_1}{1-\beta_2} \quad \text{and} \quad \text{var}(X_0) = \frac{\sigma_\varepsilon^2}{1-\beta_2^2}.$$

Hence X_0 is a random draw from the ensemble distribution. Implicitly it has been assumed that the distributions of ε and X_0 are both normal. This should have been stated explicitly.

13.4 Suppose that Y_t is determined by the process:

$$Y_t = Y_{t-1} + \varepsilon_t + \lambda\varepsilon_{t-1}$$

where ε_t is iid. Show that the process for Y_t is nonstationary unless λ takes a certain value.

Answer:

Lagging and substituting back to time 0:

$$Y_t = Y_0 + \sum_{s=1}^{t} \varepsilon_t + \lambda \sum_{s=0}^{t-1} \varepsilon_t = Y_0 + (1+\lambda) \sum_{s=1}^{t-1} \varepsilon_t + \varepsilon_t + \lambda\varepsilon_0.$$

The expectation of Y_t, taken at time 0, is Y_0 and independent of time. The variance of Y_t is $((t-1)(1+\lambda)^2 + 1 + \lambda^2)\,\sigma_\varepsilon^2$. The process is nonstationary because the variance is dependent on time, unless $\lambda = -1$, in which case the process is stationary. It reduces to:

$$Y_t = Y_0 + \varepsilon_t - \varepsilon_0.$$

The covariance between Y_t and Y_{t-s} is zero for all s greater than 0 if ε_0 is taken as predetermined. It is equal to the variance of ε if ε_0 is treated as random. Either way, it is independent of time.

13.11 Suppose that a series is generated as:

$$X_t = \beta_2 X_{t-1} + \varepsilon_t$$

with β_2 equal to $1-\delta$, where δ is small. Demonstrate that, if δ is small enough that terms involving δ^2 may be neglected, the variance may be approximated as:

$$\begin{aligned} \sigma_{X_t}^2 &= ((1-[2t-2]\delta) + \cdots + (1-2\delta) + 1)\,\sigma_\varepsilon^2 \\ &= (1-(t-1)\delta)\,t\,\sigma_\varepsilon^2 \end{aligned}$$

and draw your conclusions concerning the properties of the time series.

Answer:

$$X_t = \beta_2^t X_0 + \beta_2^{t-1}\varepsilon_1 + \cdots + \varepsilon_t.$$

Hence:

$$\begin{aligned} \sigma_{X_t}^2 &= \left(\beta_2^{2t-2} + \cdots + \beta_2^2 + 1\right)\sigma_\varepsilon^2 \\ &= \left((1-\delta)^{2t-2} + \cdots + (1-\delta)^2 + 1\right)\sigma_\varepsilon^2 \\ &= ((1-(2t-2)\delta) + \cdots + (1-2\delta) + 1)\,\sigma_\varepsilon^2 \end{aligned}$$

assuming that δ is so small that terms involving δ^2 may be neglected. (Note that the expansion of $(1+x)^n$ is $\left(1+nx+\frac{n(n-1)}{2!}x^2+\cdots\right)$ and if x is so small that terms involving x^2 and higher powers of x may be neglected, the expansion reduces to $(1+nx)$.) Thus:

$$\begin{aligned}\sigma^2_{X_t} &= (t-2\delta(t-1+\cdots+1))\,\sigma^2_\varepsilon \\ &= (t-\delta t(t-1))\,\sigma^2_\varepsilon \\ &= (1-(t-1)\delta)\,t\,\sigma^2_\varepsilon.\end{aligned}$$

It follows that, for finite t, the variance is a function of t and hence that the series exhibits nonstationary behavior for finite t, even though it is stationary.

13.15 Demonstrate that, for Case (e), Y_t is determined by:

$$Y_t = t\,\beta_1 + \frac{t(t+1)}{2}\delta + Y_0 + \sum_{s=1}^{t}\varepsilon_s.$$

This implies that the process is a convex quadratic function of time, implausible empirically.

Answer:

The simplest proof is a proof by induction. Suppose that the expression is valid for time t. Then Y_{t+1} is given by:

$$\begin{aligned}Y_t &= \beta_1 + Y_t + \delta(t+1) + \varepsilon_{t+1} \\ &= \beta_1 + \left(t\,\beta_1 + \frac{t(t+1)}{2}\delta + Y_0 + \sum_{s=1}^{t}\varepsilon_s\right) + \delta(t+1) + \varepsilon_{t+1} \\ &= (t+1)\beta_1 + \frac{(t+1)(t+2)}{2}\delta + Y_0 + \sum_{s=1}^{t+1}\varepsilon_s\end{aligned}$$

and so it is valid for time $t+1$. But it is true for time 1. So it is valid for all $t \geq 1$.

13.17 Demonstrate that the OLS estimator of δ in the model:

$$Y_t = \beta_1 + \delta t + \varepsilon_t, \qquad t = 1,\ldots,T$$

is hyperconsistent. Show also that it is unbiased in finite samples, despite the fact that Y_t is nonstationary.

Answer:

Let $\widehat{\delta}$ be the OLS estimator of δ. Following the analysis in Chapter 2, $\widehat{\delta}$ may be decomposed as:

$$\widehat{\delta} = \delta + \sum_{t=1}^{T} a_t u_t$$

where:

$$a_t = \frac{t-0.5T}{\sum\limits_{s=1}^{T}(s-0.5T)^2}.$$

Since a_t is deterministic:

$$E(\widehat{\delta}) = \delta + \sum_{t=1}^{T} a_t E(u_t) = \delta$$

and the estimator is unbiased. The variance of $\widehat{\delta}$, conditional on T, is:

$$\sigma_{\widehat{\delta}}^2 = \frac{\sigma_\varepsilon^2}{\sum_{t=1}^{T}(t - 0.5(T+1))^2}.$$

Now:

$$\begin{aligned}\sum_{t=1}^{T}\left(t - \frac{1}{2}(T+1)\right)^2 &= \sum_{t=1}^{T} t^2 - (T+1)\sum_{t=1}^{T} t + \frac{1}{4}T(T+1)^2 \\ &= \frac{1}{6}T(T+1)(2T+1) - \frac{1}{2}T(T+1)^2 + \frac{1}{4}T(T+1)^2 \\ &= \frac{T+1}{12}(4T^2 + 2T - 6T^2 - 6T + 3T^2 + 3T) \\ &= \frac{T^3 - T}{12}.\end{aligned}$$

Thus the variance is (asymptotically) inversely proportional to T^3 and the estimator is hyperconsistent.

13.6 Answers to the additional exercises

A13.1 The population series exhibits steady growth and is therefore obviously nonstationary. The growth is partly due to an excess of births over deaths and partly due to immigration. The question is whether variations in these factors are likely to be offsetting in the sense that a relatively large birth/ death excess one year is somehow automatically counterbalanced by a relatively small one in a subsequent year, or that a relatively large rate of immigration one year stimulates a reaction that leads to a relatively small one later. Such compensating mechanisms do not seem to exist, so trendstationarity may be ruled out. Population is a very good example of an integrated series with the effects of shocks being permanently incorporated in its level.

A13.2 It is difficult to come to any firm conclusion regarding this series. At first sight it looks like a random walk. On closer inspection, you will notice that after an initial decline in the first few years, the series appears to be stationary, with a high degree of correlation. The series is too short to allow one to discriminate between the two possibilities.

A13.3 As expected, given that the series is evidently nonstationary, the coefficient of $LGPOP(-1)$, -0.05, is close to zero and not significant. When we difference the series, the coefficient of $DLGPOP(-1)$ is -0.16 and not significant, even at the 5 per cent level. One possibility, which does not seem plausible, is that the population series is I(2). It is more likely that it is I(1), the first difference being stationary but highly autocorrelated.

A13.4 If the process is nonstationary, $\rho = 1$. If it is stationary, it could lie anywhere in the range $-1 < \rho < 1$. We must have a specific value for the null hypothesis. Hence we are forced to use nonstationarity as the null hypothesis, despite the inconvenience of having to compute alternative critical values of t.

A13.5 The model has been rewritten:

$$X_t - X_{t-1} = (\rho - 1)X_{t-1} + \varepsilon_t$$

so that the coefficient of X_{t-1} is zero under the null hypothesis of nonstationarity. We see that the null hypothesis is not rejected at any significance level, despite the fact that we know that the series is stationary. However, the estimate of the coefficient of X_{t-1}, -0.38, is not particularly close to zero. It implies an estimate of 0.67 for ρ, close to the actual value. This is a common outcome. Unit root tests generally have low power, making it generally difficult or impossible to discriminate between nonstationary processes and highly autocorrelated stationary processes.

A13.6 Where the hypothetical cointegrating relationship has a constant but no trend, as in the present case, the critical values of t are -3.34 and -3.90 at the 5 and 1 per cent levels, respectively (Davidson and MacKinnon, 1993). Hence the test indicates that we have a cointegrating relationship only for $DENT$ and then only at the 5 per cent level. However, one knows in advance that the residuals are likely to be highly autocorrelated. Many of the coefficients are greater than 0.2 in absolute terms and perfectly compatible with a hypothesis of highly autocorrelated stationarity.

Test of cointegration

	$\widehat{\beta}_2$	s.e.	t		$\widehat{\beta}_2$	s.e.	t
ADM	-0.09	0.06	-1.69	*GASO*	-0.08	0.05	-1.62
BOOK	-0.17	0.08	-2.24	*HOUS*	-0.31	0.12	-2.52
BUSI	-0.23	0.09	-2.40	*LEGL*	-0.26	0.10	-2.59
CLOT	-0.41	0.13	-3.17	*MAGS*	-0.39	0.13	-3.03
DENT	-0.51	0.15	-3.51	*MASS*	-0.07	0.05	-1.48
DOC	-0.35	0.12	-2.99	*OPHT*	-0.14	0.08	-1.86
FLOW	-0.22	0.10	-2.14	*RELG*	-0.17	0.07	-2.35
FOOD	-0.29	0.11	-2.61	*TELE*	-0.22	0.09	-2.35
FURN	-0.32	0.10	-3.29	*TOB*	-0.16	0.10	-1.66
GAS	-0.24	0.09	-2.79	*TOYS*	-0.17	0.09	-1.96

A13.7 The expected value of Y_t is $\beta_1 t + Y_0$, and thus it is not independent of t, one of the conditions for stationarity. Similarly for Z_t. However:

$$Y_t - \beta_1 - \beta_2 Z_t = \varepsilon_t$$

and is therefore I(0).

A13.8

$$Y_t - X_t - Z_t = W_t.$$

Since W_t is stationary, the left side of the equation is a cointegrating relationship.

A13.9 *Two series* XA_t *and* XB_t *are generated as:*

$$\begin{aligned} XA_t &= RA_t + SA_t \\ XB_t &= RB_t + SB_t. \end{aligned}$$

Explain whether it is possible for XA_t *and* XB_t *to be stationary.*

Explain whether it is possible for them to be cointegrated.

A combination of a nonstationary process and a stationary one is nonstationary. Hence both X_A and X_B are nonstationary.

Since the nonstationary components of X_A and X_B are unrelated, there is no linear combination that is stationary, and so the series are not cointegrated.

Two series YA_t *and* YB_t *are generated as*

$$\begin{aligned} YA_t &= RA_t + SA_t \\ YB_t &= RA_t + SB_t. \end{aligned}$$

Explain whether it is possible for YA_t *and* YB_t *to be cointegrated.*

$$YA_t - YB_t = SA_t - SB_t.$$

This is a cointegrating relationship for YA_t and YB_t since $SA_t - SB_t$ is stationary.

Two series ZA_t *and* ZB_t *are generated as*

$$\begin{aligned} ZA_t &= RA_t + RB_t + SA_t \\ ZB_t &= RA_t - RB_t + SB_t. \end{aligned}$$

Explain whether it is possible for ZA_t *and* ZB_t *to be stationary.*

No linear combination of RA_t and RB_t can be stationary since they are independent random walks, and so ZA_t and ZB_t are both nonstationary.

Explain whether it is possible for them to be cointegrated.

No linear combination of ZA_t and ZB_t can eliminate both RA_t and RB_t, so there is no cointegrating relationship.