Chapter 12

Properties of regression models with time series data

12.1 Overview

This chapter begins with a statement of the regression model assumptions for regressions using time series data, paying particular attention to the assumption that the disturbance term in any time period be distributed independently of the regressors in all time periods. There follows a general discussion of autocorrelation: the meaning of the term, the reasons why the disturbance term may be subject to it, and the consequences of it for OLS estimators. The chapter continues by presenting the Durbin–Watson test for AR(1) autocorrelation and showing how the problem may be eliminated. Next it is shown why OLS yields inconsistent estimates when the disturbance term is subject to autocorrelation and the regression model includes a lagged dependent variable as an explanatory variable. Then the chapter shows how the restrictions implicit in the AR(1) specification may be tested using the common factor test, and this leads to a more general discussion of how apparent autocorrelation may be caused by model misspecification. This in turn leads to a general discussion of the issues involved in model selection and, in particular, to the general-to-specific methodology.

12.2 Learning outcomes

After working through the corresponding chapter in the text, studying the corresponding slideshows, and doing the starred exercises in the text and the additional exercises in this subject guide, you should be able to:

- explain the concept of autocorrelation and the difference between positive and negative autocorrelation
- describe how the problem of autocorrelation may arise
- describe the consequences of autocorrelation for OLS estimators, their standard errors, and t and F tests, and how the consequences change if the model includes a lagged dependent variable
- perform the Breusch–Godfrey and Durbin–Watson d tests for autocorrelation
- explain how the problem of AR(1) autocorrelation may be eliminated
- describe the restrictions implicit in the AR(1) specification

- perform the common factor test
- explain how apparent autocorrelation may arise as a consequence of the omission of an important variable or the mathematical misspecification of the regression model
- demonstrate that the static, AR(1), and ADL(1,0) specifications are special cases of the ADL(1,1) model
- explain the principles of the general-to-specific approach to model selection and the defects of the specific-to-general approach.

12.3 Additional exercises

A12.1 The output shows the result of a logarithmic regression of expenditure on food on income, relative price, and population, using an AR(1) specification. Compare the results with those in Exercise A11.1.

```
============================================================
Dependent Variable: LGFOOD
Method: Least Squares
Sample(adjusted): 1960 2003
Included observations: 44 after adjusting endpoints
Convergence achieved after 14 iterations
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C          2.945983   3.943913   0.746969   0.4596
       LGDPI        0.469216   0.118230   3.968687   0.0003
     LGPRFOOD      -0.361862   0.122069  -2.964413   0.0052
       LGPOP        0.072193   0.379563   0.190200   0.8501
       AR(1)        0.880631   0.092512   9.519085   0.0000
============================================================
R-squared           0.996695    Mean dependent var 6.030691
Adjusted R-squared  0.996356    S.D. dependent var 0.216227
S.E. of regression  0.013053    Akaike info criter-5.732970
Sum squared resid   0.006645    Schwarz criterion -5.530221
Log likelihood      131.1253    F-statistic        2940.208
Durbin--Watson stat   1.556480    Prob(F-statistic)  0.000000
============================================================
Inverted AR Roots          .88
============================================================
```

A12.2 Perform Breusch–Godfrey and Durbin–Watson tests for autocorrelation for the logarithmic regression in Exercise A11.2. If you reject the null hypothesis of no autocorrelation, run the regression again using an AR(1) specification, and compare the results with those in Exercise A11.2.

A12.3 Perform an OLS ADL(1,1) logarithmic regression of expenditure on your category on current income, price, and population and lagged expenditure, income, price, and population. Use the results to perform a common factor test of the validity of the AR(1) specification in Exercise A12.2.

A12.4 A researcher has annual data on *LIFE*, aggregate consumer expenditure on life insurance, *DPI*, aggregate disposable personal income, and *PRELLIFE*, a price index for the cost of life insurance relative to general inflation, for the United States for the period 1959–1994. *LIFE* and *DPI* are measured in US$ billion. *PRELLIFE* is an index number series with 1992 = 100. She defines *LGLIFE*, *LGDPI*, and *LGPRLIFE* as the natural logarithms of *LIFE*, *DPI*, and *PRELLIFE*, respectively. She fits the regressions shown in columns (1) – (4) of the table, each with *LGLIFE* as the dependent variable. (Standard errors in parentheses; OLS = ordinary least squares; AR(1) is a specification appropriate when the disturbance term follows a first-order autoregressive process; *B–G* is the Breusch–Godfrey test statistic for AR(1) autocorrelation; d = Durbin–Watson d statistic; $\widehat{\rho}$ is the estimate of the autoregressive parameter in a first-order autoregressive process.)

	(1)	(2)	(3)	(4)	(5)
	OLS	AR(1)	OLS	OLS	OLS
LGDPI	1.37	1.41	0.42	0.28	—
	(0.10)	(0.25)	(0.60)	(0.17)	
LGPRLIFE	−0.67	−0.78	−0.59	−0.26	—
	(0.35)	(0.50)	(0.51)	(0.21)	
LGLIFE(−1)	—	—	0.82	0.79	0.98
			(0.10)	(0.09)	(0.02)
LGDPI(−1)	—	—	−0.15	—	—
			(0.61)		
LGPRLIFE(−1)	—	—	0.38	—	—
			(0.53)		
constant	−4.39	−4.20	−0.50	−0.51	0.12
	(0.88)	(1.69)	(0.72)	(0.70)	(0.08)
R^2	0.958	0.985	0.986	0.986	0.984
RSS	0.2417	0.0799	0.0719	0.0732	0.0843
B–G	23.48	—	0.61	0.34	0.10
d	0.36	1.85	2.02	1.92	2.05
$\widehat{\rho}$	—	0.82	—	—	—
		(0.11)			

- Discuss whether specification (1) is an adequate representation of the data.
- Discuss whether specification (3) is an adequate representation of the data.
- Discuss whether specification (2) is an adequate representation of the data.
- Discuss whether specification (4) is an adequate representation of the data.
- If you were presenting these results at a seminar, what would you say were your conclusions concerning the most appropriate of specifications (1) – (4)?
- At the seminar a commentator points out that in specification (4) neither *LGDPI* nor *LGPRLIFE* have significant coefficients and so these variables should be dropped. As it happens, the researcher has considered this specification, and the results are shown as specification (5) in the table. What would be your answer to the commentator?

A12.5 A researcher has annual data on the yearly rate of change of the consumer price index, p, and the yearly rate of change of the nominal money supply, m, for a certain country for the 51-year period 1958–2008. He fits the following regressions, each with p as the dependent variable. The first four regressions are fitted using OLS. The fifth is fitted using a specification appropriate when the disturbance term is assumed to follow an AR(1) process. $p(-1)$ indicates p lagged one year. $m(-1)$, $m(-2)$, and $m(-3)$ indicate m lagged 1, 2, and 3 years, respectively.

(1) explanatory variable m.

(2) explanatory variables m, $m(-1)$, $m(-2)$, and $m(-3)$.

(3) explanatory variables m, $p(-1)$, and $m(-1)$.

(4) explanatory variables m and $p(-1)$.

(5) explanatory variable m.

The results are shown in the table. Standard errors are shown in parentheses. RSS is the residual sum of squares. $B-G$ is the Breusch–Godfrey test statistic for AR(1) autocorrelation. d is the Durbin–Watson d statistic.

	1	2	3	4	5
	OLS	OLS	OLS	OLS	AR(1)
m	0.95	0.50	0.40	0.18	0.90
	(0.05)	(0.30)	(0.12)	(0.09)	(0.08)
$m(-1)$	—	0.30	−0.30	—	—
		(0.30)	(0.10)		
$m(-2)$	—	−0.15	—	—	—
		(0.30)			
$m(-3)$	—	0.30	—	—	—
		(0.30)			
$p(-1)$	—	—	0.90	0.80	—
			(0.20)	(0.20)	
constant	0.05	0.04	0.06	0.05	0.06
	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)
RSS	0.0200	0.0150	0.0100	0.0120	0.0105
B–G	35.1	27.4	0.39	0.26	0.57
d	0.10	0.21	2.00	2.00	1.90

- Looking at all five regressions together, evaluate the adequacy of:
  - specification 1.
  - specification 2.
  - specification 3.
  - specification 4.
- Explain why specification 5 is a restricted version of one of the other specifications, stating the restriction, and explaining the objective of the manipulations that lead to specification 5.
- Perform a test of the restriction embodied in specification 5.
- Explain which would be your preferred specification.

A12.6 Derive the short-run (current year) and long-run (equilibrium) effect of m on p for each of the five specifications in Exercise A12.5, using the estimated coefficients.

A12.7 A researcher has annual data on aggregate consumer expenditure on taxis, *TAXI*, and aggregate disposable personal income, *DPI*, both measured in $ billion at 2000 constant prices, and a relative price index for taxis, P, equal to 100 in 2000, for the United States for the period 1981–2005.

Defining *LGTAXI*, *LGDPI*, and *LGP* as the natural logarithms of *TAXI*, *DPI*, and P, respectively, he fits regressions (1) – (4) shown in the table. OLS = ordinary least squares; AR(1) indicates that the equation was fitted using a specification appropriate for first-order autoregressive autocorrelation; $\widehat{\rho}$ is an estimate of the parameter in the AR(1) process; *B–G* is the Breusch–Godfrey statistic for AR(1) autocorrelation; d is the Durbin–Watson d statistic; standard errors are given in parentheses.

	(1)	(2)	(3)	(4)
	OLS	AR(1)	OLS	AR(1)
LGDPI	2.06	1.28	2.28	2.24
	(0.10)	(0.84)	(0.05)	(0.07)
LGP	—	—	−0.99	−0.97
			(0.09)	(0.11)
constant	−12.75	−7.45	−9.58	−9.45
	(0.68)	(5.89)	(0.40)	(0.54)
$\widehat{\rho}$	—	0.88	—	0.26
		(0.09)		(0.22)
B–G	17.84	—	1.47	—
d	0.31	1.40	1.46	1.88
R^2	0.95	0.98	0.99	0.99

Figure 12.1 shows the actual values of *LGTAXI* and the fitted values from regression (1). Figure 12.2 shows the residuals from regression (1) and the values of *LGP*.

- Evaluate regression (1).
- Evaluate regression (2). Explain mathematically what assumptions were being made by the researcher when he used the AR(1) specification and why he hoped the results would be better than those obtained with regression (1).
- Evaluate regression (3).
- Evaluate regression (4). In particular, discuss the possible reasons for the differences in the standard errors in regressions (3) and (4).
- At a seminar one of the participants says that the researcher should consider adding lagged values of *LGTAXI*, *LGDPI*, and *LGP* to the specification. What would be your view?

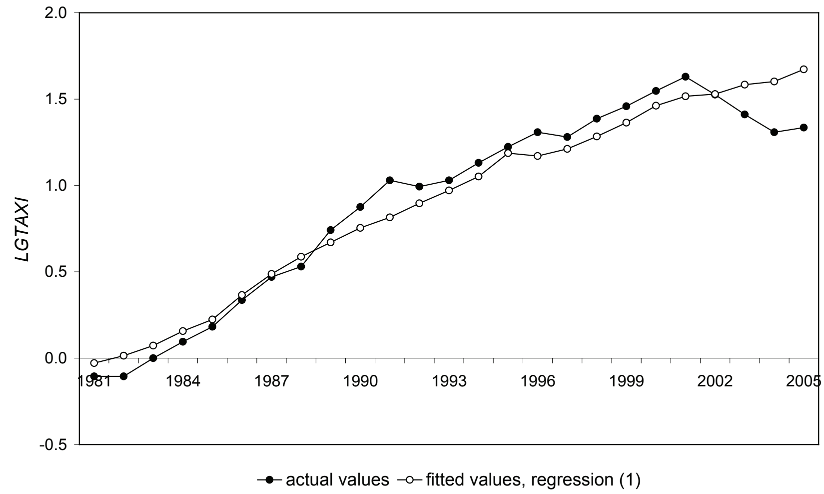


Figure 12.1: Actual values of $LGTAXI$ and the fitted values from regression (1).

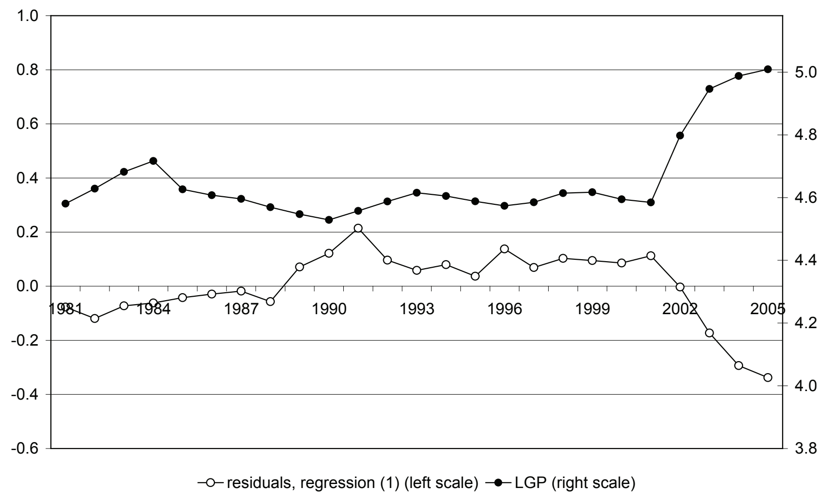


Figure 12.2: Residuals from regression (1) and the values of LGP.

A12.8 A researcher has annual data on I, investment as a percentage of gross domestic product, and r, the real long-term rate of interest for a certain economy for the period 1981–2010. He regresses I on r, (1) using ordinary least squares (OLS), (2) using an estimator appropriate for AR(1) residual autocorrelation, and (3) using OLS but adding $I(-1)$ and $r(-1)$ (I and r lagged one time period) as explanatory variables. The results are shown in columns (1), (2), and (3) of the table below. The residuals from regression (1) are shown in Figure 12.3.

He then obtains annual data on g, the rate of growth of gross domestic product of the economy, for the same period, and repeats the regressions, adding g (and, where appropriate, $g(-1)$) to the specifications as an explanatory variable. The results are shown in columns (4), (5), and (6) of the table. r and g are measured as per cent per year. The data for g are plotted in the figure.

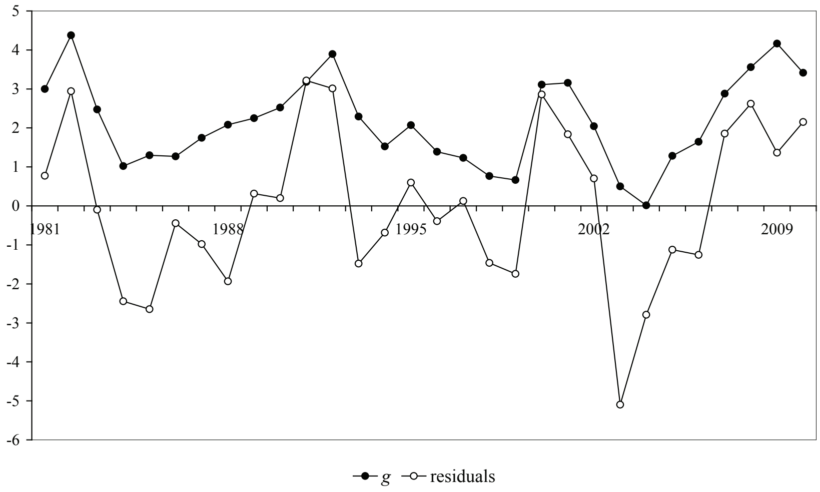


Figure 12.3: Residuals from regression (1).

	OLS	AR(1)	OLS	OLS	AR(1)	OLS
	(1)	(2)	(3)	(4)	(5)	(6)
r	−0.87	−0.83	−0.87	−1.81	−1.88	−1.71
	(0.98)	(1.05)	(1.08)	(0.49)	(0.50)	(0.52)
$I(-1)$	—	—	0.37	—	—	−0.22
			(0.16)			(0.18)
$r(-1)$	—	—	0.64	—	—	−0.98
			(1.08)			(0.64)
g	—	—	—	1.61	1.61	1.92
				(0.17)	(0.18)	(0.20)
$g(-1)$	—	—	—	—	—	−0.02
						(0.33)
$\widehat{\rho}$	—	0.37	—	—	−0.16	—
		(0.18)			(0.20)	
Constant	9.31	9.21	4.72	9.26	9.54	13.24
	(3.64)	(3.90)	(4.48)	(1.77)	(1.64)	(2.69)
B–G	4.42	—	4.24	0.70	—	0.98
d	0.99	1.36	1.33	2.30	2.05	2.09
RSS	120.5	103.9	103.5	27.4	26.8	23.5

Note: standard errors are given in parentheses. $\widehat{\rho}$ is the estimate of the autocorrelation parameter in the AR(1) specification. B–G is the Breusch–Godfrey statistic for AR(1) autocorrelation. d is the Durbin–Watson d statistic.

- Explain why the researcher was not satisfied with regression (1).
- Evaluate regression (2). Explain why the coefficients of $I(-1)$ and $r(-1)$ are not reported, despite the fact that they are part of the regression specification.
- Evaluate regression (3).

- Evaluate regression (4).
- Evaluate regression (5).
- Evaluate regression (6).
- Summarise your conclusions concerning the evaluation of the different regressions. Explain whether an examination of the figure supports your conclusions

A12.9 In Exercise A11.5 you performed a test of a restriction. The result of this test will have been invalidated if you found that the specification was subject to autocorrelation. How should the test be performed, assuming the correct specification is ADL(1,1)?

A12.10 Given data on a univariate process:

$$Y_t = \beta_1 + \beta_2 y_{t-1} + u_t$$

where $|\beta_2| < 1$ and u_t is iid, the usual OLS estimators will be consistent but subject to finite-sample bias. How should the model be fitted if u_t is subject to an AR(1) process?

A12.11 Explain what is correct, incorrect, confused or incomplete in the following statements, giving a brief explanation if not correct.

- The disturbance term in a regression model is said to be autocorrelated if its values in a sample of observations are not distributed independently of each other.
- When the disturbance term is subject to autocorrelation, the ordinary least squares estimators are inefficient and inconsistent, but they are not biased, and the t tests are invalid.
- It is a common problem in time series models because it always occurs when the dependent variable is correlated with its previous values.
- If this is the case, it could be eliminated by including the lagged value of the dependent variable as an explanatory variable.
- However, if the model is correctly specified and the disturbance term satisfies the regression model assumptions, adding the lagged value of the dependent variable as an explanatory variable will have the opposite effect and cause the disturbance term to be autocorrelated.
- A second way of dealing with the problem of autocorrelation is to use an instrumental variable.
- If the autocorrelation is of the AR(1) type, randomising the order of the observations will cause the Breusch–Godfrey statistic to be near zero, and the Durbin–Watson statistic to be near 2, thereby eliminating the problem.

12.4 Answers to the starred exercises in the textbook

12.7 Prove that σ_u^2 is related to σ_ε^2 as shown in (12.31), and show that weighting the first observation by $\sqrt{1-\rho^2}$ eliminates the heteroskedasticity.

Answer:

(12.31) is:

$$\sigma_u^2 = \frac{1}{1-\rho^2}\sigma_\varepsilon^2$$

and it assumes the first order AR(1) process (12.26): $u_t = \rho u_{t-1} + \varepsilon_t$. From the AR(1) process, neglecting transitory effects, $\sigma_{u_t} = \sigma_{u_{t-1}} = \sigma_u$ and so:

$$\sigma_u^2 = \rho^2\sigma_u^2 + \sigma_\varepsilon^2 = \frac{1}{1-\rho^2}\sigma_\varepsilon^2.$$

(Note that the covariance between u_{t-1} and ε_t is zero.) If the first observation is weighted by $\sqrt{1-\rho^2}$, the variance of the disturbance term will be:

$$\left(\sqrt{1-\rho^2}\right)^2\sigma_u^2 = (1-\rho^2)\frac{1}{1-\rho^2}\sigma_\varepsilon^2 = \sigma_\varepsilon^2$$

and it will therefore be the same as in the other observations in the sample.

12.10 The table gives the results of three logarithmic regressions using the Cobb–Douglas data for Y_t, K_t, and L_t, index number series for real output, real capital input, and real labor input, respectively, for the manufacturing sector of the United States for the period 1899–1922, reproduced in Exercise 11.6 (method of estimation as indicated; standard errors in parentheses; d = Durbin–Watson d statistic; B–G = Breusch–Godfrey test statistic for first-order autocorrelation):

	1: OLS	2: AR(1)	3: OLS
$\log K$	0.23	0.22	0.18
	(0.06)	(0.07)	(0.56)
$\log L$	0.81	0.86	1.03
	(0.15)	(0.16)	(0.15)
$\log Y(-1)$	—	—	0.40
			(0.21)
$\log K(-1)$	—	—	0.17
			(0.51)
$\log L(-1)$	—	—	−1.01
			(0.25)
constant	−0.18	−0.35	1.04
	(0.43)	(0.51)	(0.41)
$\widehat{\rho}$	—	0.19	—
		(0.25)	
R^2	0.96	0.96	0.98
RSS	0.0710	0.0697	0.0259
d	1.52	1.54	1.46
B–G	0.36	—	1.54

The first regression is that performed by Cobb and Douglas. The second fits the same specification, allowing for AR(1) autocorrelation. The third specification uses OLS with lagged variables. Evaluate the three regression specifications.

Answer:

For the first specification, the Breusch–Godfrey LM test for autocorrelation yields statistics of 0.36 (first order) and 1.39 (second order), both satisfactory. For the Durbin–Watson test, d_L and d_U are 1.19 and 1.55 at the 5 per cent level and 0.96 and 1.30 at the 1 per cent level, with 24 observations and two explanatory variables. Hence the specification appears more or less satisfactory. Fitting the model with an AR(1) specification makes very little difference, the estimate of ρ being low. However, when we fit the general ADL(1,1) model, neither of the first two specifications appears to be an acceptable simplification. The F statistic for dropping all the lagged variables is:

$$F(3,18) = \frac{(0.0710 - 0.0259)/3}{0.0259/18} = 10.45.$$

The critical value of $F(3,18)$ at the 0.1 per cent level is 8.49. The common factor test statistic is:

$$23 \log \frac{0.0697}{0.0259} = 22.77$$

and the critical value of chi-squared with two degrees of freedom is 13.82 at the 0.1 per cent level. The Breusch–Godfrey statistic for first-order autocorrelation is 1.54.

We come to the conclusion that Cobb and Douglas, who actually fitted a restricted version of the first specification, imposing constant returns to scale, were a little fortunate to obtain the plausible results they did.

12.11 Derive the final equation in Box 12.2 from the first two equations in the box. What assumptions need to be made when fitting the model?

Answer:

This exercise overlaps Exercise 11.17. The first two equations in the box are:

$$\begin{aligned} Y_t &= \beta_1 + \beta_2 X^e_{t+1} + u_t \\ X^e_{t+1} - X^e_t &= \lambda(X_t - X^e_t). \end{aligned}$$

We can rewrite the second equation as:

$$X^e_{t+1} = \lambda X_t + (1-\lambda)X^e_t.$$

Substituting this into the first equation, we have:

$$Y_t = \beta_1 + \beta_2\lambda X_t + \beta_2(1-\lambda)X^e_t + u_t.$$

This includes the unobservable X^e_t on the right side. However, lagging the second equation, we have:

$$X^e_t = \lambda X_{t-1} + (1-\lambda)X^e_{t-1}.$$

Hence:

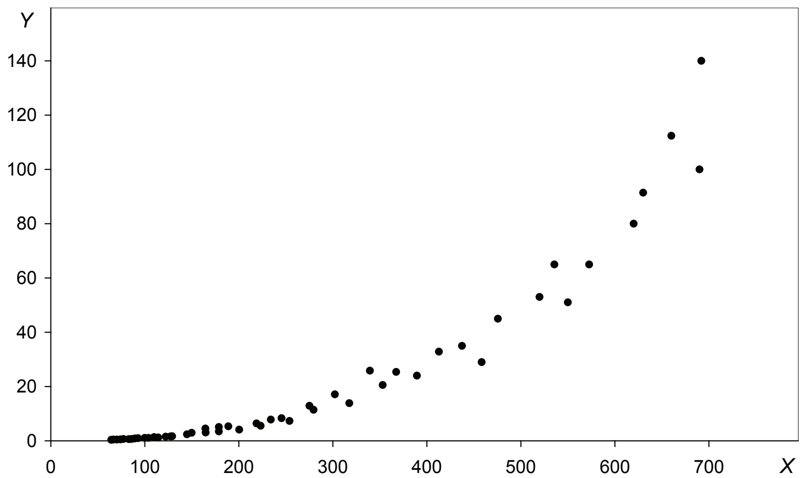
$$Y_t = \beta_1 + \beta_2\lambda X_t + \beta_2\lambda(1-\lambda)X_{t-1} + \beta_2(1-\lambda)^2 X^e_{t-1} + u_t.$$

This includes the unobservable X^e_{t-1} on the right side. However, continuing to lag and substitute, we have:

$$Y_t = \beta_1 + \beta_2\lambda X_t + \beta_2\lambda(1-\lambda)X_{t-1} + \cdots + \beta_2\lambda(1-\lambda)^s X_{t-s} + \beta_2(1-\lambda)^{s+1}X^e_{t-s} + u_t.$$

Provided that s is large enough for $\beta_2(1-\lambda)^{s+1}$ to be very small, this may be fitted, omitting the unobservable final term, with negligible omitted variable bias. We would fit it with a nonlinear regression technique that respected the constraints implicit in the theoretical structure of the coefficients. The disturbance term is unaffected by the manipulations. Hence it is sufficient to assume that it is well-behaved in the original specification.

12.14 Using the 50 observations on two variables Y and X shown in the diagram below, an investigator runs the following five regressions (estimation method as indicated; standard errors in parentheses; all variables as logarithms in the logarithmic regressions; d = Durbin–Watson d statistic; B–G = Breusch–Godfrey test statistic):



	1	2	3	4	5
	Linear		Logarithmic		
	OLS	AR(1)	OLS	AR(1)	OLS
X	0.16	0.03	2.39	2.39	1.35
	(0.01)	(0.05)	(0.03)	(0.03)	(0.70)
$Y(-1)$	—	—	—	—	−0.11
					(0.15)
$X(-1)$	—	—	—	—	1.30
					(0.75)
$\widehat{\rho}$	—	1.16	—	−0.14	—
		(0.06)		(0.15)	
constant	−21.88	−2.52	−11.00	−10.99	−12.15
	(3.17)	(8.03)	(0.15)	(0.14)	(1.67)
R^2	0.858	0.974	0.993	0.993	0.993
RSS	7663	1366	1.011	0.993	0.946
d	0.26	2.75	2.17	1.86	21.95
B–G	39.54	—	0.85	—	1.03

Discuss each of the five regressions, explaining which is your preferred specification.

Answer:

The scatter diagram reveals that the relationship is nonlinear. If it is fitted with a linear regression, the residuals must be positive for the largest and smallest values of X and negative for the middle ones. As a consequence it is no surprise to find a high Breusch–Godfrey statistic, above 10.83, the critical value of $\chi^2(1)$ at the 0.1% level, and a low Durbin–Watson statistic, below 1.32, the critical value at the 1 per cent level. Equally it is no surprise to find that an AR(1) specification does not yield satisfactory results, the Durbin–Watson statistic now indicating negative autocorrelation.

By contrast the logarithmic specification appears entirely satisfactory, with a Breusch–Godfrey statistic of 0.85 and a Durbin–Watson statistic of 1.82 (d_U is 1.59 at the 5 per cent level). Comparing it with the ADL(1,1) specification, the F statistic for dropping the lagged variables is:

$$F(2,46) = \frac{(1.084 - 1.020)/2}{1.020/46} = 1.44.$$

The critical value of $F(2,40)$ at the 5 per cent level is 3.23. Hence we conclude that specification (3) is an acceptable simplification. Specifications (4) and (5) are inefficient, and this accounts for their larger standard errors.

12.15 Using the data on food in the Demand Functions data set, the following regressions were run, each with the logarithm of food as the dependent variable: (1) an OLS regression on a time trend T defined to be 1 in 1959, 2 in 1960, etc., (2) an AR(1) regression using the same specification, and (3) an OLS regression on T and the logarithm of food lagged one time period, with the results shown in the table (standard errors in parentheses).

	1: OLS	2: AR(1)	3: OLS
T	0.0181	0.0166	0.0024
	(0.0005)	(0.0021)	(0.0016)
$LGFOOD(-1)$	—	—	0.8551
			(0.0886)
constant	5.7768	5.8163	0.8571
	(0.0106)	(0.0586)	(0.5101)
$\widehat{\rho}$	—	0.8551	—
		(0.0886)	
R^2	0.9750	0.9931	0.9931
RSS	0.0327	0.0081	0.0081
d	0.2752	1.3328	1.3328
h	—	—	2.32

Discuss why each regression specification appears to be unsatisfactory. Explain why it was not possible to perform a common factor test.

Answer:

The Durbin–Watson statistic in regression (1) is very low, suggesting AR(1) autocorrelation. However, it remains below 1.40, d_L for a 5 per cent significance test with one explanatory variable and 35 observations, in the AR(1) specification in regression (2). The reason of course is that the model is very poorly specified, with two obvious major variables, income and price, excluded.

With regard to the impossibility of performing a common factor test, suppose that the original model is written:

$$LGFOOD_t = \beta_1 + \beta_2 T + u_t.$$

Lagging the model and multiplying through by ρ, we have:

$$\rho LGFOOD_{t-1} = \beta_1\rho + \beta_2\rho(T-1) + \rho u_{t-1}.$$

Subtracting and rearranging, we obtain the AR(1) specification:

$$\begin{aligned} LGFOOD_t &= \beta_1(1-\rho) + \rho LGFOOD_{t-1} + \beta_2 T - \beta_2\rho(T-1) + u_t - \rho u_{t-1} \\ &= \beta_1(1-\rho) + \beta_2\rho + \rho LGFOOD_{t-1} + \beta_2(1-\rho)T + \varepsilon_t. \end{aligned}$$

However, this specification does not include any restrictions. The coefficient of $LGFOOD_{t-1}$ provides an estimate of ρ. The coefficient of T then provides an estimate of β_2. Finally, given these estimates, the intercept provides an estimate of β_1. The AR(1) and ADL(1,1) specifications are equivalent in this model, the reason being that the variable $(T-1)$ is merged into T and the intercept.

12.5 Answers to the additional exercises

A12.1 The Durbin–Watson statistic in the OLS regression is 0.49, causing us to reject the null hypothesis of no autocorrelation at the 1 per cent level. The Breusch–Godfrey statistic (not shown) is 25.12, also causing the null hypothesis of no autocorrelation to be rejected at a high significance level. Apart from a more satisfactory Durbin–Watson statistic, the results for the AR(1) specification are similar to those of the OLS one. The income and price elasticities are a little larger. The estimate of the population elasticity, negative in the OLS regression, is now effectively zero, suggesting that the direct effect of population on expenditure on food is offset by a negative income effect. The standard errors are larger than those for the OLS regression, but the latter are invalidated by the autocorrelation and therefore should not be taken at face value.

A12.2 All of the regressions exhibit strong evidence of positive autocorrelation. The Breusch–Godfrey test statistic for AR(1) autocorrelation is above the critical value of 10.82 (critical value of chi-squared with one degree of freedom at the 0.1% significance level) and the Durbin–Watson d statistic is below 1.20 (d_L, 1 per cent level, 45 observations, $k = 4$). The Durbin–Watson statistics for the AR(1) specification are generally much more healthy than those for the OLS one, being scattered around 2.

Breusch–Godfrey and Durbin–Watson statistics, logarithmic OLS regression including population					
	B–G	*d*		*B–G*	*d*
ADM	19.37	0.683	*GASO*	36.21	0.212
BOOK	25.85	0.484	*HOUS*	23.88	0.523
BUSI	24.31	0.507	*LEGL*	24.30	0.538
CLOT	18.47	0.706	*MAGS*	19.27	0.667
DENT	14.02	0.862	*MASS*	21.97	0.612
DOC	24.74	0.547	*OPHT*	31.64	0.328
FLOW	24.13	0.535	*RELG*	26.30	0.497
FOOD	24.95	0.489	*TELE*	30.08	0.371
FURN	22.92	0.563	*TOB*	27.84	0.421
GAS	23.41	0.569	*TOYS*	20.04	0.668

Since autocorrelation does not give rise to bias, one would not expect to see systematic changes in the point estimates of the coefficients. However, since multicollinearity is to some extent a problem for most categories, the coefficients do exhibit greater volatility than is usual when comparing OLS and AR(1) results. Fortunately, most of the major changes seem to be for the better. In particular, some implausibly high income elasticities are lower. Likewise, the population elasticities are a little less erratic, but most are still implausible, with large standard errors that reflect the continuing underlying problem of multicollinearity.

AR(1) logarithmic regression										
	LGDPI		*LGP*		*LGPOP*		$\widehat{\rho}$		R^2	*d*
	coef.	s.e.	coef.	s.e.	coef.	s.e.	coef.	s.e.		
ADM	−0.34	0.34	0.00	0.20	3.73	0.95	0.76	0.08	0.992	2.03
BOOK	0.46	0.41	−1.06	0.29	2.73	1.25	0.82	0.10	0.990	1.51
BUSI	0.43	0.24	0.19	0.25	2.45	0.70	0.69	0.10	0.997	1.85
CLOT	1.07	0.16	−0.56	0.15	−0.49	0.71	0.84	0.08	0.999	2.19
DENT	1.14	0.18	−1.01	0.15	0.69	0.73	0.56	0.13	0.996	1.86
DOC	0.85	0.25	−0.30	0.26	1.26	0.77	0.83	0.10	0.997	1.61
FLOW	0.71	0.41	−1.04	0.44	0.74	1.33	0.78	0.09	0.994	1.97
FOOD	0.47	0.12	−0.36	0.12	0.07	0.38	0.88	0.09	0.997	1.56
FURN	1.73	0.36	−0.37	0.51	−1.62	1.55	0.92	0.06	0.994	2.00
GAS	−0.02	0.34	0.01	0.08	0.29	0.97	0.83	0.06	0.933	2.12
GASO	0.75	0.15	−0.14	0.03	−0.64	0.48	0.93	0.04	0.998	1.65
HOUS	0.27	0.08	−0.27	0.09	−0.03	0.54	0.98	0.00	0.997	1.66
LEGL	0.89	0.20	−0.19	0.22	−0.54	0.80	0.77	0.10	0.989	1.90
MAGS	0.98	0.30	−1.24	0.39	−0.23	0.92	0.73	0.12	0.983	1.73
MASS	0.06	0.28	−0.72	0.11	1.31	0.97	0.94	0.04	0.944	1.95
OPHT	1.99	0.60	−0.92	0.97	−1.45	1.85	0.90	0.08	0.991	1.67
RELG	0.86	0.18	−1.15	0.26	2.00	0.56	0.66	0.10	0.999	2.08
TELE	0.70	0.20	−0.56	0.13	2.44	0.71	0.87	0.10	0.999	1.51
TOB	0.38	0.22	−0.35	0.07	−0.99	0.66	0.79	0.10	0.960	2.37
TOYS	0.89	0.18	−0.58	0.13	1.61	0.66	0.75	0.12	0.999	1.77

A12.3 The table gives the residual sum of squares for the unrestricted ADL(1,1) specification and that for the restricted AR(1) one, the fourth column giving the chi-squared statistic for the common factor test.

Before performing the common factor test, one should check that the ADL(1,1) specification is itself free from autocorrelation using the Breusch–Godfrey test. The fifth column gives the *B–G* statistic for AR(1) autocorrelation. All but one of the statistics are below the critical value at the 5 per cent level, 3.84. The exception is that for *LEGL*. It should be remembered that the Breusch–Godfrey test is a large-sample tests and in this application, with only 44 observations, the sample is rather small.

Common factor test and tests of autocorrelation for ADL(1,1) model

	$RSS_{\text{ADL}(1,2)}$	$RSS_{\text{AR}(1)}$	Chi-squared	*B–G*
ADM	0.029792	0.039935	12.89	0.55
BOOK	0.070478	0.086240	8.88	1.25
BUSI	0.032074	0.032703	0.85	0.57
CLOT	0.009097	0.010900	7.96	1.06
DENT	0.019281	0.021841	5.49	1.22
DOC	0.025598	0.028091	4.09	0.33
FLOW	0.084733	0.084987	0.13	0.01
FOOD	0.005562	0.006645	7.83	3.12
FURN	0.050880	0.058853	6.41	0.29
GAS	0.035682	0.045433	10.63	0.66
GASO	0.006898	0.009378	13.51	2.91
HOUS	0.001350	0.002249	22.46	0.77
LEGL	0.026650	0.034823	11.77	8.04
MAGS	0.043545	0.051808	7.64	0.03
MASS	0.029125	0.033254	5.83	0.15
OPHT	0.139016	0.154629	4.68	0.08
RELG	0.013910	0.014462	1.71	0.32
TELE	0.014822	0.017987	8.52	0.97
TOB	0.021403	0.021497	0.19	3.45
TOYS	0.015313	0.015958	1.82	2.60

For the common factor test, the critical values of chi-squared are 7.81 and 11.34 at the 5 and 1 per cent levels, respectively, with 3 degrees of freedom. Summarising the results, we find:

- AR(1) specification not rejected: *BUSI*, *DENT*, *DOC*, *FLOW*, *FURN*, *MAGS*, *MASS*, *OPHT*, *RELG*, *TOB*, *TOYS*.
- AR(1) specification rejected at 5 per cent level: *BOOK*, *CLOT*, *FOOD*, *GAS*, *TELE*.
- AR(1) specification rejected at 1 per cent level: *ADM*, *GASO*, *HOUS*, *LEGL*.

A12.4 *Discuss whether specification (1) is an adequate representation of the data.*

The Breusch–Godfrey statistic is well in excess of the critical value at the 0.1 per cent significance level, 10.83. Likewise, the Durbin–Watson statistic is far below

1.15, d_L at the 1 per cent level with two explanatory variables and 36 observations. There is therefore strong evidence of either severe AR(1) autocorrelation or some serious misspecification.

Discuss whether specification (3) is an adequate representation of the data.

The only item that we can check is whether it is free from autocorrelation. The Breusch–Godfrey statistic is well under 3.84, the critical value at the 5 per cent significance level, and so there is no longer evidence of autocorrelation or misspecification.

Discuss whether specification (2) is an adequate representation of the data.

Let the original model be written:

$$\begin{aligned} LGLIFE &= \beta_1 + \beta_2 LGDPI + \beta_3 LGDPRLIFE + u \\ u_t &= \rho u_{t-1} + \varepsilon_t. \end{aligned}$$

The AR(1) specification is then:

$$\begin{aligned} LGLIFE &= \beta_1(1-\rho) + \rho LGLIFE(-1) + \beta_2 LGDPI - \beta_2\rho LGDPI(-1) \\ &\quad + \beta_3 LGDPRLIFE - \beta_3\rho LGPRLIFE(-1) + \varepsilon_t. \end{aligned}$$

This is a restricted version of the ADL(1,1) model because it incorporates nonlinear restrictions on the coefficients of $LGDPI(-1)$ and $LGPRLIFE(-1)$. In the ADL(1,1) specification, minus the product of the coefficients of $LGLIFE(-1)$ and $LGDPI$ is $-0.82 \times 0.42 = -0.34$. The coefficient of $LGDPI(-1)$ is smaller than this, but then its standard error is large. Minus the product of the coefficients of $LGLIFE(-1)$ and $LGPRLIFE$ is $-0.82 \times -0.59 = 0.48$. The coefficient of $LGPRLIFE(-1)$ is fairly close, bearing in mind that its standard error is also large. The coefficient of $LGLIFE(-1)$ is exactly equal to the estimate of ρ in the AR(1) specification.

The common factor test statistic is:

$$35 \log_e \frac{0.799}{0.719} = 3.69.$$

The null hypothesis is that the two restrictions are valid. Under the null hypothesis, the test statistic has a chi-squared distribution with 2 degrees of freedom. Its critical value at the 5 per cent level is 5.99. Hence we do not reject the restrictions and the AR(1) specification therefore does appear to be acceptable.

Discuss whether specification (4) is an adequate representation of the data.

We note that $LGLDPI(-1)$ and $LGPRLIFE(-1)$ do not have significant t statistics, but since they are being dropped simultaneously, we should perform an F test of their joint explanatory power:

$$F(2, 29) = \frac{(0.732 - 0.719)/2}{0.719/29} = 0.26.$$

Since this is less than 1, it is not significant at any significance level and so we do not reject the null hypothesis that the coefficients of $LGLDPI(-1)$ and

$LGPRLIFE(-1)$ are both 0. Hence it does appear that we can drop these variables. We should also check for autocorrelation. The Breusch–Godfrey statistic indicates that there is no problem.

If you were presenting these results at a seminar, what would you say were your conclusions concerning the most appropriate of specifications (1) – (4)?

There is no need to mention (1). (3) is not a candidate because we have found acceptable simplifications that are likely to yield more efficient parameter estimates , and this is reflected in the larger standard errors compared with (2) and (4). We cannot discriminate between (2) and (4).

At the seminar a commentator points out that in specification (4) neither LGDPI nor LGPRLIFE have significant coefficients and so these variables should be dropped. As it happens, the researcher has considered this specification, and the results are shown as specification (5) in the table. What would be your answer to the commentator?

Comparing (3) and (5):

$$F(4,29) = \frac{(0.843 - 0.719)/4}{0.719/29} = 1.25.$$

The critical value of $F(4,29)$ at the 5 per cent level is 2.70, so it would appear that the joint explanatory power of the 4 income and price variables is not significant. However, it does not seem sensible to drop current income and current price from the model. The reason that they have so little explanatory power is that the short-run effects are small, life insurance being subject to long-term contracts and thus a good example of a category of expenditure with a large amount of inertia. The fact that income in the AR(1) specification has a highly significant coefficient is concrete evidence that it should not be dropped.

A12.5 *Looking at all five regressions together, evaluate the adequacy of:*

- *specification 1.*
- *specification 2.*
- *specification 3.*
- *specification 4.*

- Specification 1 has a very high Breusch–Godfrey statistic and a very low Durbin–Watson statistic. There is evidence of either severe autocorrelation or model misspecification.
- Specification 2 also has a very high Breusch–Godfrey statistic and a very low Durbin–Watson statistic. Further, there is evidence of multicollinearity: large standard errors (although comparisons are very dubious given low DW), and implausible coefficients.
- Specification 3 seems acceptable. In particular, there is no evidence of autocorrelation since the Breusch–Godfrey statistic is low.
- Specification 4: dropping $m(-1)$ may be expected to cause omitted variable bias since the t statistic for its coefficient was -3.0 in specification 3.

(Equivalently, the F statistic is:

$$F(1,46) = \frac{(0.0120 - 0.0100)/1}{0.0100/46} = 0.2 \times 46 = 9.2$$

the square of the t statistic and similarly significant.)

Explain why specification 5 is a restricted version of one of the other specifications, stating the restriction, and explaining the objective of the manipulations that lead to specification 5.

Write the original model and AR(1) process:

$$\begin{aligned} p_t &= \beta_1 + \beta_2 m_t + u_t \\ u_y &= \rho u_{t-1} + \varepsilon_t. \end{aligned}$$

Then fitting:

$$p_t = \beta_1(1-\rho) + \rho p_{t-1} + \beta_2 m_t - \beta_2 \rho m_{t-1} + \varepsilon_t$$

removes the autocorrelation. This is a restricted version of specification 3, with restriction that the coefficient of m_{t-1} is equal to minus the product of the coefficients of m_t and p_{t-1}.

Perform a test of the restriction embodied in specification 5.

Comparing specifications 3 and 5, the common factor test statistic is:

$$n \log_e \left(\frac{RSS_R}{RSS_U} \right) = 50 \log \left(\frac{0.0105}{0.0100} \right) = 50 \log 1.05 \cong 50 \times 0.05 = 2.5.$$

Under the null hypothesis that the restriction implicit in the specification is valid, the test statistic is distributed as chi-squared with one degree of freedom. The critical value at the 5 per cent significance level is 3.84, so we do not reject the restriction. Accordingly, specification 5 appears to be an adequate representation of the data.

Explain which would be your preferred specification.

Specifications (3) and (5) both appear to be adequate representations of the data. (5) should yield more efficient estimators of the parameters because, exploiting an apparently-valid restriction, it is less susceptible to multicollinearity, and this appears to be confirmed by the lower standard errors.

A12.6 The models are:

1. $p_t = \beta_1 + \beta_2 m_t + u_t$
2. $p_t = \beta_1 + \beta_2 m_t + \beta_3 m_{t-1} + \beta_4 m_{t-2} + \beta_5 m_{t-3} + u_t$
3. $p_t = \beta_1 + \beta_2 m_t + \beta_3 m_{t-1} + \beta_6 p_{t-1} + u_t$
4. $p_t = \beta_1 + \beta_2 m_t + \beta_6 p_{t-1} + u_t$
5. $p_t = \beta_1(1-\beta_6) + \beta_6 p_{t-1} + \beta_2 m_t - \beta_2\beta_6 m_{t-1} + \varepsilon_t$ (writing $\rho = \beta_6$).

Hence we obtain the following estimates of $\partial p_t / \partial m_t$:

1. 0.95
2. 0.50
3. 0.40
4. 0.18
5. 0.90.

Putting p and m equal to equilibrium values, and ignoring the disturbance term, we have:

1. $\bar{p} = \beta_1 + \beta_2 \overline{m}$
2. $\bar{p} = \beta_1 + (\beta_2 + \beta_3 + \beta_4)\overline{m}$
3. $\bar{p} = \frac{1}{1-\beta_6}(\beta_1 + (\beta_2 + \beta_3)\overline{m})$
4. $\bar{p} = \frac{1}{1-\beta_6}(\beta_1 + \beta_2 \overline{m})$
5. $\bar{p} = \beta_1 + \beta_2 \overline{m}$.

Hence we obtain the following estimates of $\mathrm{d}\bar{p}/\mathrm{d}\overline{m}$:

1. 0.95
2. 0.95
3. 1.00
4. 0.90
5. 0.90.

A12.7 *Evaluate regression (1).*

Regression (1) has a very high Breusch–Godfrey statistic and a very low Durbin–Watson statistic. The null hypothesis of no autocorrelation is rejected at the 1 per cent level for both tests. Alternatively, the test statistics might indicate some misspecification problem.

Evaluate regression (2). Explain mathematically what assumptions were being made by the researcher when he used the AR(1) specification and why he hoped the results would be better than those obtained with regression (1).

Regression (2) has been run on the assumption that the disturbance term follows an AR(1) process:

$$u_t = \rho u_{t-1} + \varepsilon_t.$$

On the assumption that the regression model should be:

$$LGTAXI_t = \beta_1 + \beta_2 LGDPI_t + u_t,$$

the autocorrelation can be eliminated in the following way: lag the regression model by one time period and multiply through by ρ:

$$\rho LGTAXI_{t-1} = \beta_1 \rho + \beta_2 \rho LGDPI_{t-1} + \rho u_{t-1}.$$

Subtract this from the regression model:

$$LGTAXI_t - \rho LGTAXI_{t-1} = \beta_1(1-\rho) + \beta_2 LGDPI_t - \beta_2 \rho LGDPI_{t-1} + u_t - \rho u_{t-1}.$$

Hence one obtains a specification free from autocorrelation:

$$LGTAXI_t = \beta_1(1-\rho) + \rho LGTAXI_{t-1} + \beta_2 LGDPI_t - \beta_2\rho LGDPI_{t-1} + \varepsilon_t.$$

The Durbin–Watson statistic is still low, suggesting that fitting the AR(1) specification was an inappropriate response to the problem.

Evaluate regression (3).

In regression (3) the Breusch–Godfrey statistic suggests that, for this specification, there is not a problem of autocorrelation (the Durbin–Watson statistic is indecisive). This suggests that the apparent autocorrelation in the regression (1) is in fact attributable to the omission of the price variable.

This is corroborated by the diagrams, which show that large negative residuals occurred when the price rose and positive ones when it fell. The effect is especially obvious in the final years of the sample period.

Evaluate regression (4). In particular, discuss the possible reasons for the differences in the standard errors in regressions (3) and (4).

In regression (4), the Durbin–Watson statistic does not indicate a problem of autocorrelation. Overall, there is little to choose between regressions (3) and (4). It is possible that there was some autocorrelation in regression (3) and that it has been rectified by using AR(1) in regression (4). It is also possible that autocorrelation was not actually a problem in regression (3). Regressions (3) and (4) yield similar estimates of the income and price elasticities and in both cases the elasticities are significantly different from zero at a high significance level. If regression (4) is the correct specification, the lower standard errors in regression (3) should be disregarded because they are invalid. If regression (3) is the correct specification, AR(1) estimation will yield inefficient estimates; which could account for the higher standard errors in regression (4).

At a seminar one of the participants says that the researcher should consider adding lagged values of LGTAXI, LGDPI, and LGP to the specification. What would be your view?

Specifications (2) and (4) already contain the lagged values, with restrictions on the coefficients of $LGDPI(-1)$ and $LGP(-1)$.

A12.8 *Explain why the researcher was not satisfied with regression (1).*

The researcher was not satisfied with the results of regression (1) because the Breusch–Godfrey statistic was 4.42, above the critical value at the 5 per cent level, 3.84, and because the Durbin–Watson d statistic was only 0.99. The critical value of d_L with one explanatory variable and 30 observations is 1.35. Thus there is evidence that the specification may be subject to autocorrelation.

Evaluate regression (2). Explain why the coefficients of I(-1) and r(-1) are not reported, despite the fact that they are part of the regression specification.

Specification (2) is equally unsatisfactory. The fact that the Durbin–Watson statistic has remained low is an indication that the reason for the low d in (1) was not an AR(1) disturbance term. RSS is very high compared with those in specifications (4) – (6). The coefficient of $I(-1)$ is not reported as such because it is the estimate $\widehat{\rho}$. The coefficient of $r(-1)$ is not reported because it is constrained to be minus the product of $\widehat{\rho}$. and the coefficient of I.

Evaluate regression (3).

Specification (3) is the unrestricted ADL(1,1) model of which the previous AR(1) model was a restricted version and it suffers from the same problems. There is still evidence of positive autocorrelation, since the Breusch–Godfrey statistic, 4.24, is high and RSS is still much higher than in the three remaining specifications.

Evaluate regression (4).

Specification (4) seems fine. The null hypothesis of no autocorrelation is not rejected by either the Breusch–Godfrey statistic or the Durbin–Watson statistic. The coefficients are significant and have the expected signs.

Evaluate regression (5).

The AR(1) specification (5) does not add anything because there was no evidence of autocorrelation in (4). The estimate of ρ is not significantly different from zero.

Evaluate regression (6).

Specification (6) does not add anything either. t tests on the coefficients of the lagged variables indicate that they are individually not significantly different from zero. Likewise the joint hypothesis that their coefficients are all equal to zero is not rejected by an F test comparing RSS in (4) and (6):

$$F(3,23) = \frac{(27.4 - 23.5)/3}{23.5/23} = 1.27.$$

The critical value of $F(3,23)$ at the 5 per cent level is 3.03. [There is no point in comparing (5) and (6) using a common factor test, but for the record the test statistic is:

$$n \log_e \frac{RSS_R}{RSS_U} = 29 \log_e \frac{26.8}{23.5} = 3.81.$$

The critical value of chi-squared with 2 degrees of freedom at the 5 per cent level is 5.99.]

Summarise your conclusions concerning the evaluation of the different regressions. Explain whether an examination of the figure supports your conclusions.

The overall conclusion is that the static model (4) is an acceptable representation of the data and the apparent autocorrelation in specifications (1) – (3) is attributable to the omission of g. Figure 12.3 shows very clearly that the residuals in specification (1) follow the same pattern as g, confirming that the apparent autocorrelation in the residuals is in fact attributable to the omission of g from the specification.

A12.9 *In Exercise A11.5 you performed a test of a restriction. The result of this test will have been invalidated if you found that the specification was subject to autocorrelation. How should the test be performed, assuming the correct specification is ADL(1,1)?*

If the ADL(1,1) model is written:

$$\begin{aligned}\log CAT &= \beta_1 + \beta_2 \log DPI + \beta_3 \log P + \beta_4 \log POP + \beta_5 \log CAT_{-1} \\ &\quad + \beta_6 log DPI_{-1} + \beta_7 \log P_{-1} + \beta_8 \log POP_{-1} + u\end{aligned}$$

the restricted version with expenditure per capita a function of income per capita is:

$$\begin{aligned}\log \frac{CAT}{POP} &= \beta_1 + \beta_2 \log \frac{DPI}{POP} + \beta_3 \log P + \beta_5 \log \frac{CAT_{-1}}{POP_{-1}} \\ &\quad + \beta_6 \log \frac{DPI_{-1}}{POP_{-1}} + \beta_7 \log P_{-1} + u.\end{aligned}$$

Comparing the two equations, we see that the restrictions are $\beta_4 = 1 - \beta_2$ and $\beta_8 = -\beta_5 - \beta_6$. The usual F statistic should be constructed and compared with the critical values of $F(2, 28)$.

A12.10 Let the AR(1) process be written:

$$u_t = \rho u_{t-1} + \varepsilon_t.$$

As the specification stands, OLS would yield inconsistent estimates because both the explanatory variable and the disturbance term depend on u_{t-1}. Applying the standard procedure, multiplying the lagged relationship by ρ and subtracting, one has:

$$Y_t - \rho Y_{t-1} = \beta_1(1-\rho) + \beta_2 Y_{t-1} - \beta_2 \rho Y_{t-1} + u_t - \rho u_{t-1}.$$

Hence:

$$Y_t = \beta_1(1-\rho) + (\beta_2 + \rho) Y_{t-1} - \beta_2 \rho Y_{t-2} + \varepsilon_t.$$

It follows that the model should be fitted as a second-order, rather than as a first-order, process. There are no restrictions on the coefficients. OLS estimators will be consistent, but subject to finite-sample bias.

A12.11 *Explain what is correct. incorrect, confused or incomplete in the following statements, giving a brief explaination if not correct.*

- *The disturbance term in a regression model is said to be autocorrelated if its values in a sample of observations are not distributed independently of each other.*

  Correct.

- *When the disturbance term is subject to autocorrelation, the ordinary least squares estimators are inefficient ...*

  Correct.

- *...and inconsistent...*

  Incorrect, unless there is a lagged dependent variable.

- *...but they are not biased...*

  Correct, unless there is a lagged dependent variable.

- *…and the t tests are invalid.*

  Correct.

- *It is a common problem in time series models because it always occurs when the dependent variable is correlated with its previous values.*

  Incorrect.

- *If this is the case, it could be eliminated by including the lagged value of the dependent variable as an explanatory variable.*

  In general, incorrect. However, a model requiring a lagged dependent variable could appear to exhibit autocorrelation if the lagged dependent variable were omitted, and including it could eliminate the apparent problem.

- *However, if the model is correctly specified and the disturbance term satisfies the regression model assumptions, adding the lagged value of the dependent variable as an explanatory variable will have the opposite effect and cause the disturbance term to be autocorrelated.*

  Nonsense.

- *A second way of dealing with the problem of autocorrelation is to use an instrumental variable.*

  More nonsense.

- *If the autocorrelation is of the AR(1) type, randomising the order of the observations will cause the Durbin–Watson statistic to be near 2…*

  Correct.

- *…thereby eliminating the problem.*

  Incorrect. The problem will have been disguised, not rectified.