Chapter 11
Models using time series data

11.1 Overview

This chapter introduces the application of regression analysis to time series data, beginning with static models and then proceeding to dynamic models with lagged variables used as explanatory variables. It is shown that multicollinearity is likely to be a problem in models with unrestricted lag structures and that this provides an incentive to use a parsimonious lag structure, such as the Koyck distribution. Two models using the Koyck distribution, the adaptive expectations model and the partial adjustment model, are described, together with well-known applications to aggregate consumption theory, Friedman's permanent income hypothesis in the case of the former and Brown's habit persistence consumption function in the case of the latter. The chapter concludes with a discussion of prediction and stability tests in time series models.

11.2 Learning outcomes

After working through the corresponding chapter in the text, studying the corresponding slideshows, and doing the starred exercises in the text and the additional exercises in this subject guide, you should be able to:

- explain why multicollinearity is a common problem in time series models, especially dynamic ones with lagged explanatory variables
- describe the properties of a model with a lagged dependent variable (ADL(1,0) model)
- describe the assumptions underlying the adaptive expectations and partial adjustment models
- explain the properties of OLS estimators of the parameters of ADL(1,0) models
- explain how predetermined variables may be used as instruments in the fitting of models using time series data
- explain in general terms the objectives of time series analysts and those constructing VAR models.

11.3 Additional exercises

A11.1 The output below shows the result of linear and logarithmic regressions of expenditure on food on income, relative price, and population (measured in thousands) using the Demand Functions data set, together with the correlations among the variables. Provide an interpretation of the regression coefficients and perform appropriate statistical tests.

```
============================================================
Dependent Variable: FOOD
Method: Least Squares
Sample: 1959 2003
Included observations: 45
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C         -19.49285   88.86914  -0.219343   0.8275
        DPI         0.031713   0.010658   2.975401   0.0049
     PRELFOOD       0.403356   0.365133   1.104681   0.2757
        POP         0.001140   0.000563   2.024017   0.0495
============================================================
R-squared           0.988529    Mean dependent var 422.0374
Adjusted R-squared  0.987690    S.D. dependent var 91.58053
S.E. of regression  10.16104    Akaike info criteri7.559685
Sum squared resid   4233.113    Schwarz criterion  7.720278
Log likelihood     -166.0929    F-statistic        1177.745
Durbin-Watson stat  0.404076    Prob(F-statistic)  0.000000
============================================================
```

```
============================================================
Dependent Variable: LGFOOD
Method: Least Squares
Sample: 1959 2003
Included observations: 45
============================================================
       Variable    Coefficient Std. Error t-Statistic  Prob.
============================================================
          C         5.293654   2.762757   1.916077   0.0623
        LGDPI       0.589239   0.080158   7.351014   0.0000
       LGPRFOOD    -0.122598   0.084355  -1.453361   0.1537
        LGPOP      -0.289219   0.258762  -1.117706   0.2702
============================================================
R-squared           0.992245    Mean dependent var 6.021331
Adjusted R-squared  0.991678    S.D. dependent var 0.222787
S.E. of regression  0.020324    Akaike info criter-4.869317
Sum squared resid   0.016936    Schwarz criterion -4.708725
Log likelihood      113.5596    F-statistic        1748.637
Durbin-Watson stat  0.488502    Prob(F-statistic)  0.000000
============================================================
```

```
                     Correlation Matrix
============================================================
              LGFOOD      LGDPI      LGPRFOOD     LGPOP
============================================================
   LGFOOD     1.000000    0.995896   -0.613437    0.990566
   LGDPI      0.995896    1.000000   -0.604658    0.995241
  LGPRFOOD   -0.613437   -0.604658    1.000000   -0.641226
   LGPOP      0.990566    0.995241   -0.641226    1.000000
============================================================
```

A11.2 Perform regressions parallel to those in Exercise A11.1 using your category of expenditure and provide an interpretation of the coefficients.

A11.3 The output shows the result of a logarithmic regression of expenditure on food per capita, on income per capita, both measured in US$ million, and the relative price index for food. Provide an interpretation of the coefficients, demonstrate that the specification is a restricted version of the logarithmic regression in Exercise A11.1, and perform an F test of the restriction.

```
============================================================
Dependent Variable: LGFOODPC
Method: Least Squares
Sample: 1959 2003
Included observations: 45
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C          -5.425877   0.353655  -15.34231   0.0000
      LGDPIPC        0.280229   0.014641   19.14024   0.0000
     LGPRFOOD        0.052952   0.082588   0.641160   0.5249
============================================================
R-squared            0.927348    Mean dependent var-6.321984
Adjusted R-squared   0.923889    S.D. dependent var 0.085249
S.E. of regression   0.023519    Akaike info criter-4.597688
Sum squared resid    0.023232    Schwarz criterion -4.477244
Log likelihood       106.4480    F-statistic        268.0504
Durbin-Watson stat   0.417197    Prob(F-statistic)  0.000000
============================================================
```

A11.4 Perform a regression parallel to that in Exercise A11.3 using your category of expenditure. Provide an interpretation of the coefficients, and perform an F test of the restriction.

A11.5 The output shows the result of a logarithmic regression of expenditure on food per capita, on income per capita, the relative price index for food, and population. Provide an interpretation of the coefficients, demonstrate that the specification is equivalent to that for the logarithmic regression in Exercise A11.1, and use it to perform a t test of the restriction in Exercise A11.3.

```
============================================================
Dependent Variable: LGFOODPC
Method: Least Squares
Sample: 1959 2003
```

```
Included observations: 45
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
        C            5.293654   2.762757    1.916077   0.0623
      LGDPIPC        0.589239   0.080158    7.351014   0.0000
     LGPRFOOD       -0.122598   0.084355   -1.453361   0.1537
       LGPOP        -0.699980   0.179299   -3.903973   0.0003
============================================================
R-squared            0.947037    Mean dependent var-6.321984
Adjusted R-squared   0.943161    S.D. dependent var 0.085249
S.E. of regression   0.020324    Akaike info criter-4.869317
Sum squared resid    0.016936    Schwarz criterion -4.708725
Log likelihood       113.5596    F-statistic        244.3727
Durbin-Watson stat   0.488502    Prob(F-statistic)  0.000000
============================================================
```

A11.6 Perform a regression parallel to that in Exercise A11.5 using your category of expenditure, and perform a t test of the restriction implicit in the specification in Exercise A11.4.

A11.7 In Exercise 11.9 you fitted the model:

$$LGCAT = \beta_1 + \beta_2 LGDPI + \beta_3 LGDPI(-1) + \beta_4 LGPRCAT + \beta_5 LGPRCAT(-1) + u$$

where CAT stands for your category of expenditure.

- Show that $(\beta_2 + \beta_3)$ and $(\beta_4 + \beta_5)$ are theoretically the long-run (equilibrium) income and price elasticities.
- Reparameterise the model and fit it to obtain direct estimates of these long-run elasticities and their standard errors.
- Confirm that the estimates are equal to the sum of the individual shortrun elasticities found in Exercise 11.9.
- Compare the standard errors with those found in Exercise 11.9 and state your conclusions.

A11.8 In a certain bond market, the demand for bonds, B_t, in period t is negatively related to the expected interest rate, i^e_{t+1}, in period $t+1$:

$$B_t = \beta_1 + \beta_2 i^e_{t+1} + u_t \qquad (1)$$

where u_t is a disturbance term not subject to autocorrelation. The expected interest rate is determined by an adaptive expectations process:

$$i^e_{t+1} - i^e_t = \lambda(i_t - i^e_t) \qquad (2)$$

where i_t is the actual rate of interest in period t. A researcher uses the following model to fit the relationship:

$$B_t = \gamma_1 + \gamma_2 i_t + \gamma_3 B_{t-1} + v_t \qquad (3)$$

where v_t is a disturbance term.

- Show how this model may be derived from the demand function and the adaptive expectations process.
- Explain why inconsistent estimates of the parameters will be obtained if equation (3) is fitted using ordinary least squares (OLS). (A mathematical proof is not required. Do not attempt to derive expressions for the bias.)
- Describe a method for fitting the model that would yield consistent estimates.
- Suppose that u_t was subject to the first-order autoregressive process:

$$u_t = \rho u_{t-1} + \varepsilon_t$$

  where ε_t is not subject to autocorrelation. How would this affect your answer to the second part of this question?
- Suppose that the true relationship was actually:

$$B_t = \beta_1 + \beta_2 i_t + u_t \qquad (1*)$$

  with u_t not subject to autocorrelation, and the model is fitted by regressing B_t on i_t and B_{t-1}, as in equation (3), using OLS. How would this affect the regression results?
- How plausible do you think an adaptive expectations process is for modelling expectations in a bond market?

A11.9 The output shows the result of a logarithmic regression of expenditure on food on income, relative price, population, and lagged expenditure on food using the Demand Functions data set. Provide an interpretation of the regression coefficients, paying attention to both short-run and long-run dynamics, and perform appropriate statistical tests.

```
============================================================
Dependent Variable: LGFOOD
Method: Least Squares
Sample(adjusted): 1960 2003
Included observations: 44 after adjusting endpoints
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           1.487645   2.072156   0.717921   0.4771
       LGDPI         0.143829   0.090334   1.592194   0.1194
     LGPRFOOD       -0.095749   0.061118  -1.566613   0.1253
       LGPOP        -0.046515   0.189453  -0.245524   0.8073
    LGFOOD(-1)       0.727290   0.113831   6.389195   0.0000
============================================================
R-squared            0.995886    Mean dependent var 6.030691
Adjusted R-squared   0.995464    S.D. dependent var 0.216227
S.E. of regression   0.014564    Akaike info criter-5.513937
Sum squared resid    0.008272    Schwarz criterion -5.311188
Log likelihood       126.3066    F-statistic        2359.938
Durbin-Watson stat   1.103102    Prob(F-statistic)  0.000000
============================================================
```

A11.10 Perform a regression parallel to that in Exercise A11.9 using your category of expenditure. Provide an interpretation of the coefficients, and perform appropriate statistical tests.

A11.11 In his classic study *Distributed Lags and Investment Analysis* (1954), Koyck investigated the relationship between investment in railcars and the volume of freight carried on the US railroads using data for the period 1884–1939. Assuming that the desired stock of railcars in year t depended on the volume of freight in year $t-1$ and year $t-2$ and a time trend, and assuming that investment in railcars was subject to a partial adjustment process, he fitted the following regression equation using OLS (standard errors and constant term not reported):

$$\widehat{I_t} = 0.077F_{t-1} + 0.017F_{t-2} - 0.0033t - 0.110K_{t-1} \qquad R^2 = 0.85$$

where $I_t = K_t - K_{t-1}$ is investment in railcars in year t (thousands), K_t is the stock of railcars at the end of year t (thousands), and F_t is the volume of freight handled in year t (ton-miles).

Provide an interpretation of the equation and describe the dynamic process implied by it. (Note: It is best to substitute $K_t - K_{t-1}$ for I_t in the regression and treat it as a dynamic relationship determining K_t.)

A11.12 Two researchers agree that a model consists of the following relationships:

$$\begin{aligned} Y_t &= \alpha_1 + \alpha_2 X_t + u_t && (1) \\ X_t &= \beta_1 + \beta_2 Y_{t-1} + v_t && (2) \\ Z_t &= \gamma_1 + \gamma_2 Y_t + \gamma_3 X_t + \gamma_4 Q_t + w_t && (3) \end{aligned}$$

where u_t, v_t, and w_t, are disturbance terms that are drawn from fixed distributions with zero mean. It may be assumed that they are distributed independently of Q_t and of each other and that they are not subject to autocorrelation. All the parameters may be assumed to be positive and it may be assumed that $\alpha_2\beta_2 < 1$.

- One researcher asserts that consistent estimates will be obtained if (2) is fitted using OLS and (1) is fitted using IV, with Y_{t-1} as an instrument for X_t. Determine whether this is true.
- The other researcher asserts that consistent estimates will be obtained if both (1) and (2) are fitted using OLS, and that the estimate of β_2 will be more efficient than that obtained using IV. Determine whether this is true.

11.4 Answers to the starred exercises in the textbook

11.6

Year	Y	K	L	Year	Y	K	L
1899	100	100	100	1911	153	216	145
1900	101	107	105	1912	177	226	152
1901	112	114	110	1913	184	236	154
1902	122	122	118	1914	169	244	149
1903	124	131	123	1915	189	266	154
1904	122	138	116	1916	225	298	182
1905	143	149	125	1917	227	335	196
1906	152	163	133	1918	223	366	200
1907	151	176	138	1919	218	387	193
1908	126	185	121	1920	231	407	193
1909	155	198	140	1921	179	417	147
1910	159	208	144	1922	240	431	161

Source: Cobb and Douglas (1928)

The table gives the data used by Cobb and Douglas (1928) to fit the original Cobb–Douglas production function:

$$Y_t = \beta_1 K_t^{\beta_2} L_t^{\beta_3} v_t$$

Y_t, K_t, and L_t, being index number series for real output, real capital input, and real labour input, respectively, for the manufacturing sector of the United States for the period 1899–1922 (1899 = 100). The model was linearised by taking logarithms of both sides and the following regressions was run (standard errors in parentheses):

$$\begin{aligned}\widehat{\log Y} &= \underset{(0.43)}{-0.18} + \underset{(0.06)}{0.23}\log K + \underset{(0.15)}{0.81}\log L \qquad R^2 = 0.96\end{aligned}$$

Provide an interpretation of the regression coefficients.

Answer:

The elasticities of output with respect to capital and labour are 0.23 and 0.81, respectively, both coefficients being significantly different from zero at very high significance levels. The fact that the sum of the elasticities is close to one suggests that there may be constant returns to scale. Regressing output per worker on capital per worker, one has:

$$\widehat{\log \frac{Y}{L}} = \underset{(0.02)}{0.01} + \underset{(0.04)}{0.25}\log \frac{K}{L} \qquad R^2 = 0.63$$

The smaller standard error of the slope coefficient suggests a gain in efficiency. Fitting a reparameterised version of the unrestricted model:

$$\widehat{\log \frac{Y}{L}} = \underset{(0.43)}{-0.18} + \underset{(0.06)}{0.23}\log \frac{K}{L} + \underset{(0.09)}{0.04}\log L \qquad R^2 = 0.64$$

we find that the restriction is not rejected.

11.7 The Cobb–Douglas model in Exercise 11.6 makes no allowance for the possibility that output may be increasing as a consequence of technical progress, independently of K and L. Technical progress is difficult to quantify and a common way of allowing for it in a model is to include an exponential time trend:

$$Y_t = \beta_1 K_t^{\beta_2} L_t^{\beta_3} \mathrm{e}^{\rho t} v_t$$

where ρ is the rate of technical progress and t is a time trend defined to be 1 in the first year, 2 in the second, etc. The correlations between $\log K$, $\log L$ and t are shown in the table. Comment on the regression results.

$$\widehat{\log Y} = \underset{(1.38)}{2.81} - \underset{(0.34)}{0.53} \log K + \underset{(0.14)}{0.91} \log L + \underset{(0.021)}{0.047} t \qquad R^2 = 0.97$$

Correlation

	LGK	LGL	TIME
LGK	1.000000	0.909562	0.996834
LGL	0.909562	1.000000	0.896344
TIME	0.996834	0.896344	1.000000

Answer:

The elasticity of output with respect to labour is higher than before, now implausibly high given that, under constant returns to scale, it should measure the share of wages in output. The elasticity with respect to capital is negative and nonsensical. The coefficient of time indicates an annual exponential growth rate of 4.7 per cent, holding K and L constant. This is unrealistically high for the period in question. The implausibility of the results, especially those relating to capital and time (correlation 0.997), may be attributed to multicollinearity.

11.16 Demonstrate that the dynamic process (11.18) implies the long-run relationship given by (11.15).

Answer:

Equations (11.15) and (11.18) are:

$$\tilde{Y} = \frac{\beta_1}{1-\beta_3} + \frac{\beta_2}{1-\beta_3}\tilde{X} \tag{11.15}$$

$$\begin{aligned} Y_t &= \beta_1(1+\beta_3+\beta_3^2+\cdots) + \beta_2 X_t + \beta_2\beta_3 X_{t-1} + \beta_2\beta_3^2 X_{t-2} + \cdots \\ &\quad + u_t + \beta_3 u_{t-1} + \beta_3^2 u_{t-2} + \cdots . \end{aligned} \tag{11.18}$$

Putting $X = \tilde{X}$ for all X in (11.18), and ignoring the disturbance terms, the long-run relationship between Y and X is given by:

$$\begin{aligned} \tilde{Y} &= \beta_1(1+\beta_3+\beta_3^2+\cdots) + \beta_2\tilde{X} + \beta_2\beta_3\tilde{X} + \beta_2\beta_3^2\tilde{X} + \cdots \\ &= \frac{\beta_1}{1-\beta_3} + (1+\beta_3+\beta_3^2+\cdots)\beta_2\tilde{X} \\ &= \frac{\beta_1}{1-\beta_3} + \frac{\beta_2}{1-\beta_3}\tilde{X}. \end{aligned}$$

11.17 The compound disturbance term in the adaptive expectations model (11.37) does potentially give rise to a problem that will be discussed in Chapter 12 when we come to the topic of autocorrelation. It can be sidestepped by representing the model in the alternative form.

$$Y_t = \beta_1 + \beta_2\lambda X_t + \beta_2\lambda(1-\lambda)X_{t-1} + \cdots + \beta_2\lambda(1-\lambda)^s X_{t-s} + \beta_2(1-\lambda)^{s+1}X^e_{t-s} + u_t.$$

Show how this form might be obtained, and discuss how it might be fitted.

Answer:

We start by reprising equations (11.31) – (11.34) in the text. We assume that the dependent variable Y_t is related to X^e_{t+1}, the value of X anticipated in the next time period:

$$Y_t = \gamma_1 + \gamma_2 X^e_{t+1} + u_t. \qquad (11.31)$$

To make the model operational, we hypothesise that expectations are updated in response to the discrepancy between what had been anticipated for the current time period, X^e_{t+1}, and the actual outcome, X_t:

$$X^e_{t+1} - X^e_t = \lambda(X_t - X^e_t) \qquad (11.32)$$

where λ may be interpreted as a speed of adjustment. We can rewrite this as (11.33):

$$X^e_{t+1} = \lambda X_t + (1-\lambda)X^e_t. \qquad (11.33)$$

Hence we obtain (11.34):

$$Y_t = \gamma_1 + \gamma_2\lambda X_t + \gamma_2(1-\lambda)X^e_t + u_t. \qquad (11.34)$$

This includes the unobservable X^e_t on the right side. However, lagging (11.33), we have:

$$X^e_t = \lambda X_{t-1} + (1-\lambda)X^e_{t-1}.$$

Hence:

$$Y_t = \gamma_1 + \gamma_2\lambda X_t + \gamma_2\lambda(1-\lambda)X_{t-1} + \gamma_2(1-\lambda)^2 X^e_{t-1} + u_t.$$

This includes the unobservable X^e_{t-1} on the right side. However, continuing to lag and substitute, we have:

$$Y_t = \gamma_1 + \gamma_2\lambda X_t + \gamma_2\lambda(1-\lambda)X_{t-1} + \cdots + \gamma_2\lambda(1-\lambda)^s X_{t-s} + \gamma_2(1-\lambda)^{s+1}X^e_{t-s} + u_t.$$

Provided that s is large enough for $\gamma_2(1-\lambda)^{s+1}$ to be very small, this may be fitted, omitting the unobservable final term, with negligible omitted variable bias.

We would fit it with a nonlinear regression technique that respected the constraints implicit in the theoretical structure of the coefficients.

11.19 The output below shows the result of fitting the model:

$$\begin{aligned} LGFOOD &= \beta_1 + \beta_2\lambda LGDPI + \beta_2\lambda(1-\lambda)LGDPI(-1) + \beta_2\lambda(1-\lambda)^2 LGDPI(-2) \\ &\quad + \beta_2\lambda(1-\lambda)^3 LGDPI(-3) + \beta_3 LGPRFOOD + u \end{aligned}$$

using the data on expenditure on food in the Demand Functions data set. *LGFOOD* and *LGPRFOOD* are the logarithms of expenditure on food and the relative price index series for food. C(1), C(2), C(3), and C(4) are estimates of β_1, β_2, λ and β_3, respectively. Explain how the regression equation could be interpreted as an adaptive expectations model and discuss the dynamics implicit in it, both short-run and long-run. Should the specification have included further lagged values of $LGDPI$?

```
============================================================
Dependent Variable: LGFOOD
Method: Least Squares
Sample(adjusted): 1962 2003
Included observations: 42 after adjusting endpoints
Convergence achieved after 25 iterations
LGFOOD=C(1)+C(2)*C(3)*LGDPI + C(2)*C(3)*(1-C(3))*LGDPI(-1) + C(2)
        *C(3)*(1-C(3))^2*LGDPI(-2) + C(2)*C(3)*(1-C(3))^3*LGDPI(-3) +
        C(4)*LGPRFOOD
============================================================
                   Coefficient Std. Error t-Statistic  Prob.
============================================================
       C(1)           2.339513   0.468550   4.993091   0.0000
       C(2)           0.496425   0.012264   40.47818   0.0000
       C(3)           0.915046   0.442851   2.066264   0.0457
       C(4)          -0.089681   0.083250  -1.077247   0.2882
============================================================
R-squared             0.989621    Mean dependent var 6.049936
Adjusted R-squared    0.988802    S.D. dependent var 0.201706
S.E. of regression    0.021345    Akaike info criter-4.765636
Sum squared resid     0.017313    Schwarz criterion -4.600143
Log likelihood        104.0784    Durbin-Watson stat 0.449978
============================================================
```

Answer:

Suppose that the model is:

$$LGFOOD_t = \gamma_1 + \gamma_2 LGDPI^e_{t+1} + \gamma_3 LGPRFOOD_t + u_t$$

where $LGDPI^e_{t+1}$ is expected $LGDPI$ at time $t+1$, and that expectations for income are subject to the adaptive expectations process:

$$LGDPI^e_{t+1} - LGDPI^e_t = \lambda(LGDPI_t - LGDPI^e_t).$$

The adaptive expectations process may be rewritten:

$$LGDPI^e_{t+1} = \lambda LGDPI_t + (1-\lambda)LGDPI^e_t.$$

Lagging this equation one period and substituting, one has:

$$LGDPI^e_{t+1} = \lambda LGDPI_t + \lambda(1-\lambda)LGDPI_{t-1} + (1-\lambda)^2 LGDPI^e_{t-1}.$$

Lagging a second time and substituting, one has:

$$LGDPI^e_{t+1} = \lambda LGDPI_t + \lambda(1-\lambda)LGDPI_{t-1} + \lambda(1-\lambda)^2 LGDPI_{t-2} + (1-\lambda)^3 LGDPI^e_{t-2}.$$

Lagging a third time and substituting, one has:

$$\begin{aligned} LGDPI^e_{t+1} &= \lambda LGDPI_t + \lambda(1-\lambda)LGDPI_{t-1} + \lambda(1-\lambda)^2 LGDPI_{t-2} \\ &\quad + \lambda(1-\lambda)^3 LGDPI^e_{t-3} + (1-\lambda)^4 LGDPI^e_{t-3}. \end{aligned}$$

Substituting this into the model, dropping the final unobservable term, one has the regression specification as stated in the question.

The estimates imply that the short-run income elasticity is 0.50. The speed of adjustment of expectations is 0.92. Hence the long-run income elasticity is $0.50/0.92 = 0.54$. The price side of the model has been assumed to be static. The estimate of the price elasticity is -0.09. The coefficient of the dropped unobservable term is $\gamma_2(1-\lambda)^4$. Given our estimates of γ_2 and λ, its estimate is 0.0003. Hence we are justified in neglecting it.

11.22 A researcher is fitting the following supply and demand model for a certain commodity, using a sample of time series observations:

$$\begin{aligned} Q_{dt} &= \beta_1 + \beta_2 P_t + u_{dt} \\ Q_{st} &= \alpha_1 + \alpha_2 P_t + u_{st} \end{aligned}$$

where Q_{dt} is the amount demanded at time t, Q_{st} is the amount supplied, P_t is the market clearing price, and u_{dt} and u_{st} are disturbance terms that are not necessarily independent of each other. It may be assumed that the market clears and so $Q_{dt} = Q_{st}$.

- What can be said about the identification of (a) the demand equation, (b) the supply equation?
- What difference would it make if supply at time t was determined instead by price at time $t-1$? That is:

$$Q_{st} = \alpha_1 + \alpha_2 P_{t-1} + u_{st}.$$

- What difference would it make if it could be assumed that u_{dt} is distributed independently of u_{st}?

Answer:

The reduced form equation for P_t is:

$$P_t = \frac{1}{\alpha_2 - \beta_2}(\beta_1 - \alpha_1 + u_{dt} - u_{st}).$$

P_t is not independent of the disturbance term in either equation and so OLS would yield inconsistent estimates. There is no instrument available, so both equations are underidentified.

Provided that u_{dt} is not subject to autocorrelation, P_{t-1} could be used as an instrument in the demand equation. Provided that u_{st} is not subject to autocorrelation, OLS could be used to fit the second equation. It makes no difference whether or not u_{dt} is distributed independently of u_{st}.

The first equation could, alternatively, be fitted using OLS, with the variables switched. From the second equation, P_{t-1} determines Q_t, and then, given Q_t, the demand equation determines P_t:

$$P_t = \frac{1}{\beta_2}(Q_t - \beta_1 - u_{dt}).$$

The reciprocal of the slope coefficient provides a consistent estimator of β_2.

11.24 Consider the following simple macroeconomic model:

$$\begin{aligned} C_t &= \beta_1 + \beta_2 Y_t + u_{C_t} \\ I_t &= \alpha_1 + \alpha_2 (Y_t - Y_{t-1}) + u_{I_t} \\ Y_t &= C_t + I_t \end{aligned}$$

where C_t, I_t, and Y_t are aggregate consumption, investment, and income and u_{C_t} and u_{I_t} are disturbance terms. The first relationship is a conventional consumption function. The second relates investment to the change of output from the previous year. (This is known as an 'accelerator' model.) The third is an income identity. What can be said about the identification of the relationships in the model?

Answer:

The restriction on the coefficients of Y_t and Y_{t-1} in the investment equation complicates matters. A simple way of handling it is to define:

$$\Delta Y_t = Y_t - Y_{t-1}$$

and to rewrite the investment equation as:

$$I_t = \alpha_1 + \alpha_2 \Delta Y_t + u_{I_t}.$$

We now have four endogenous variables and four equations, and one exogenous variable. The consumption and investment equations are exactly identified. We would fit them using Y_{t-1} as an instrument for Y_t and ΔY_t, respectively. The other two equations are identities and do not need to be fitted.

11.5 Answers to the additional exercises

A11.1 The linear regression indicates that expenditure on food increases by \$0.032 billion for every extra \$ billion of disposable personal income (in other words, by 3.2 cents out of the marginal dollar), that it increases by \$0.403 billion for every point increase in the price index, and that it increases by \$0.001 billion for every additional thousand population. The income coefficient is significant at the 1 per cent level (ignoring problems to be discussed in Chapter 12). The positive price coefficient makes no sense (remember that the dependent variable is measured in real terms). The intercept has no plausible interpretation.

The logarithmic regression indicates that the income elasticity is 0.59 and highly significant, and the price elasticity is -0.12, not significant. The negative elasticity for population is not plausible. One would expect expenditure on food to increase in line with population, controlling for other factors, and hence, as a first approximation, the elasticity should be equal to 1. However, an increase in population, keeping income constant, would lead to a reduction in income per capita and hence to a negative income effect. Given that the income elasticity is less than 1, one would still expect a positive elasticity overall for population. At least the estimate is not significantly different from zero. In view of the high correlation, 0.995, between *LGDPI* and *LGPOP*, the negative estimate may well be a result of multicollinearity.

A11.2

OLS logarithmic regressions							
	LGDPI		*LGP*		*LGPOP*		R^2
	coef.	s.e.	coef.	s.e.	coef.	s.e.	
ADM	−1.43	0.20	−0.28	0.10	6.88	0.61	0.975
BOOK	−0.29	0.28	−1.18	0.21	4.94	0.82	0.977
BUSI	0.36	0.19	−0.11	0.27	2.79	0.51	0.993
CLOT	0.71	0.10	−0.70	0.05	0.15	0.36	0.998
DENT	1.23	0.14	−0.95	0.09	0.26	0.54	0.995
DOC	0.97	0.14	0.26	0.13	−0.27	0.52	0.993
FLOW	0.46	0.32	0.16	0.33	3.07	1.21	0.987
FOOD	0.59	0.08	−0.12	0.08	−0.29	0.26	0.992
FURN	0.36	0.28	−0.48	0.26	1.66	1.12	0.985
GAS	1.27	0.24	−0.24	0.06	−2.81	0.74	0.788
GASO	1.46	0.16	−0.10	0.04	−2.35	0.49	0.982
HOUS	0.91	0.08	−0.54	0.06	0.38	0.25	0.999
LEGL	1.17	0.16	−0.08	0.13	−1.50	0.54	0.976
MAGS	1.05	0.22	−0.73	0.44	−0.82	0.54	0.970
MASS	−1.92	0.22	−0.57	0.14	6.14	0.65	0.785
OPHT	0.30	0.45	0.28	0.59	3.68	1.40	0.965
RELG	0.56	0.13	−0.99	0.23	2.72	0.41	0.996
TELE	0.91	0.13	−0.61	0.11	1.79	0.49	0.998
TOB	0.54	0.17	−0.42	0.04	−1.21	0.57	0.883
TOYS	0.59	0.10	−0.54	0.06	2.57	0.39	0.999

The price elasticities mostly lie in the range 0 to −1, as they should, and therefore seem plausible. However the very high correlation between income and population, 0.995, has given rise to a problem of multicollinearity and as a consequence the estimates of their elasticities are very erratic. Some of the income elasticities look plausible, but that may be pure chance, for many are unrealistically high, or negative when obviously they should be positive. The population elasticities are even less convincing.

Correlations between prices, income and population					
	LGP, *LGDPI*	*LGP*, *LGPOP*		*LGP*, *LGDPI*	*LGP*, *LGPOP*
ADM	0.61	0.61	*GASO*	0.05	0.03
BOOK	0.88	0.87	*HOUS*	0.49	0.55
BUSI	0.98	0.97	*LEGL*	0.99	0.99
CLOT	−0.94	−0.96	*MAGS*	0.99	0.98
DENT	0.94	0.96	*MASS*	0.90	0.89
DOC	0.98	0.98	*OPHT*	−0.68	−0.67
FLOW	−0.93	−0.95	*RELG*	0.92	0.92
FOOD	−0.60	−0.64	*TELE*	−0.98	−0.99
FURN	−0.95	−0.97	*TOB*	0.83	0.86
GAS	0.77	0.76	*TOYS*	−0.97	−0.98

A11.3 The regression indicates that the income elasticity is 0.40 and the price elasticity 0.21, the former very highly significant, the latter significant at the 1 per cent level using a one-sided test. If the specification is:

$$\log \frac{FOOD}{POP} = \beta_1 + \beta_2 \log \frac{DPI}{POP} + \beta_3 \log PRELFOOD + u$$

it may be rewritten:

$$\log FOOD = \beta_1 + \beta_2 \log DPI + \beta_3 \log PRELFOOD + (1 - \beta_2) \log POP + u.$$

This is a restricted form of the specification in Exercise A11.2:

$$\log FOOD = \beta_1 + \beta_2 \log DPI + \beta_3 \log PRELFOOD + \beta_4 \log POP + u$$

with $\beta_4 = 1 - \beta_2$. We can test the restriction by comparing *RSS* for the two regressions:

$$F(1, 41) = \frac{(0.023232 - 0.016936)/1}{0.016936/41} = 15.24.$$

The critical value of $F(1, 40)$ at the 0.1 per cent level is 12.61. The critical value for $F(1, 41)$ must be slightly lower. Thus we reject the restriction. Since the restricted version is misspecified, our interpretation of the coefficients of this regression and the t tests are invalidated.

A11.4 Given that the critical values of $F(1, 41)$ at the 5 and 1 per cent levels are 4.08 and 7.31 respectively, the results of the F test may be summarised as follows:

- Restriction not rejected: *CLOT*, *DENT*, *DOC*, *FURN*, *HOUS*.
- Restriction rejected at the 5 per cent level: *MAGS*.
- Restriction rejected at the 1 per cent level: *ADM*, *BOOK*, *BUSI*, *FLOW*, *FOOD*, *GAS*, *GASO*, *LEGL*, *MASS*, *OPHT*, *RELG*, *TELE*, *TOB*, *TOYS*.

However, for reasons that will become apparent in the next chapter, these findings must be regarded as provisional.

Tests of a restriction				
	RSS_U	RSS_R	F	t
ADM	0.125375	0.480709	116.20	10.78
BOOK	0.223664	0.461853	43.66	6.61
BUSI	0.084516	0.167580	40.30	6.35
CLOT	0.021326	0.021454	0.25	−0.50
DENT	0.033275	0.034481	1.49	1.22
DOC	0.068759	0.069726	0.58	−0.76
FLOW	0.220256	0.262910	7.94	2.82
FOOD	0.016936	0.023232	15.24	−3.90
FURN	0.157153	0.162677	1.44	1.20
GAS	0.185578	0.300890	25.48	−5.05
GASO	0.078334	0.139278	31.90	−5.65
HOUS	0.011270	0.012106	3.04	1.74
LEGL	0.082628	0.102698	9.96	−3.16
MAGS	0.096620	0.106906	4.36	−2.09
MASS	0.143775	0.330813	53.34	7.30
OPHT	0.663413	0.822672	9.84	3.14
RELG	0.053785	0.135532	62.32	7.89
TELE	0.054519	0.080728	19.71	4.44
TOB	0.062452	0.087652	16.54	−4.07
TOYS	0.031269	0.071656	52.96	7.28

A11.5 If the specification is:

$$\log \frac{FOOD}{POP} = \beta_1 + \beta_2 \log \frac{DPI}{POP} + \beta_3 \log PRELFOOD + \gamma_1 POP + u$$

it may be rewritten:

$$\log FOOD = \beta_1 + \beta_2 \log DPI + \beta_3 \log PRELFOOD + (1 - \beta_2 + \gamma_1) \log POP + u.$$

This is equivalent to the specification in Exercise A11.1:

$$\log FOOD = \beta_1 + \beta_2 \log DPI + \beta_3 \log PRELFOOD + \beta_4 \log POP + u$$

with $\beta_4 = 1 - \beta_2 + \gamma_1$. Note that this is not a restriction. (1) – (3) are just different ways of writing the unrestricted model.

A t test of $H_0 : \gamma_1 = 0$ is equivalent to a t test of $H_0 : \beta_4 = 1 - \beta_2$, that is, that the restriction in Exercise A11.3 is valid. The t statistic for $LGPOP$ in the regression is −3.90, and hence again we reject the restriction. Note that the test is equivalent to the F test. −3.90 is the square root of 15.24, the F statistic, and it can be shown that the critical value of t is the square root of the critical value of F.

A11.6 The t statistics for all the categories of expenditure are supplied in the table in the answer to Exercise A11.4. Of course they are equal to the square root of the F statistic, and their critical values are the square roots of the critical values of F, so the conclusions are identical and, like those of the F test, should be treated as provisional.

A11.7 *Show that $\beta_2 + \beta_3$ and $(\beta_4 + \beta_5)$ are theoretically the long-run (equilibrium) income and price elasticities.*

In equilibrium, $LGCAT = \overline{LGCAT}$, $LGDPI = LGDPI(-1) = \overline{LGDPI}$ and $LGPRCAT = LGPRCAT(-1) = \overline{LGPRCAT}$. Hence, ignoring the transient effect of the disturbance term:

$$\begin{aligned}\overline{LGCAT} &= \beta_1 + \beta_2\overline{LGDPI} + \beta_3\overline{LGDPI} + \beta_4\overline{LGPRCAT} + \beta_5\overline{LGPRCAT} \\ &= \beta_1 + (\beta_2 + \beta_3)\overline{LGDPI} + (\beta_4 + \beta_5)\overline{LGPRCAT}.\end{aligned}$$

Thus the long-run equilibrium income and price elasticities are $\theta = \beta_2 + \beta_3$ and $\phi = \beta_4 + \beta_5$, respectively.

Reparameterise the model and fit it to obtain direct estimates of these long-run elasticities and their standard errors.

We will reparameterise the model to obtain direct estimates of θ and ϕ and their standard errors. Write $\beta_3 = \theta - \beta_2$ and $\phi = \beta_4 + \beta_5$ and substitute for β_3 and β_5 in the model. We obtain:

$$\begin{aligned}LGCAT &= \beta_1 + \beta_2 LGDPI + (\theta - \beta_2)LGDPI(-1) + \beta_4 LGPRCAT + (\phi - \beta_4)LGPRCAT(-1) + u \\ &= \beta_1 + \beta_2(LGDPI - LGDPI(-1)) + \theta LGDPI(-1) \\ &\quad + \beta_4(LGPRCAT - LGPRCAT(-1)) + \phi LGPRCAT(-1) + u \\ &= \beta_1 + \beta_2 DLGDPI + \theta LGDPI(-1) + \beta_4 DLGPRCAT + \phi LGPRCAT(-1) + u\end{aligned}$$

where $DLGDPI = LGDPI - LGDPI(-1)$ and $DLGPRCAT = LGPRCAT - LGPRCAT(-1)$.

The output for $HOUS$ is shown below. $DLGPRCAT$ has been abbreviated as $DLGP$.

```
============================================================
Dependent Variable: LGHOUS
Method: Least Squares
Sample(adjusted): 1960 2003
Included observations: 44 after adjusting endpoints
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           0.020785   0.144497   0.143844   0.8864
      DLGDPI         0.329571   0.150397   2.191340   0.0345
     LGDPI(-1)       1.013147   0.006815   148.6735   0.0000
       DLGP         -0.088813   0.165651  -0.536144   0.5949
   LGPRHOUS(-1)     -0.447176   0.035927  -12.44689   0.0000
============================================================
R-squared           0.999039    Mean dependent var 6.379059
Adjusted R-squared  0.998940    S.D. dependent var 0.421861
S.E. of regression  0.013735    Akaike info criter-5.631127
Sum squared resid   0.007357    Schwarz criterion -5.428379
Log likelihood      128.8848    F-statistic         10131.80
Durbin-Watson stat  0.536957    Prob(F-statistic)  0.000000
============================================================
```

Confirm that the estimates are equal to the sum of the individual shortrun elasticities found in Exercise 11.9.

The estimates of the long-run income and price elasticities are 1.01 and −0.45, respectively. The output below is for the model in its original form, where the coefficients are all short-run elasticities. It may be seen that, for both income and price, the sum of the estimates of the shortrun elasticities is indeed equal to the estimate of the long-run elasticity in the reparameterised specification.

```
============================================================
Dependent Variable: LGHOUS
Method: Least Squares
Sample(adjusted): 1960 2003
Included observations: 44 after adjusting endpoints
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C          0.020785   0.144497   0.143844   0.8864
       LGDPI        0.329571   0.150397   2.191340   0.0345
     LGDPI(-1)      0.683575   0.147111   4.646648   0.0000
     LGPRHOUS      -0.088813   0.165651  -0.536144   0.5949
   LGPRHOUS(-1)    -0.358363   0.165782  -2.161660   0.0368
============================================================
R-squared           0.999039    Mean dependent var 6.379059
Adjusted R-squared  0.998940    S.D. dependent var 0.421861
S.E. of regression  0.013735    Akaike info criter-5.631127
Sum squared resid   0.007357    Schwarz criterion -5.428379
Log likelihood      128.8848    F-statistic        10131.80
Durbin-Watson stat  0.536957    Prob(F-statistic)  0.000000
============================================================
```

Compare the standard errors with those found in Exercise 11.9 and state your conclusions.

The standard errors of the long-run elasticities in the reparameterised version are much smaller than those of the short-run elasticities in the original specification, and the t statistics accordingly much greater. Our conclusion is that it is possible to obtain relatively precise estimates of the long-run impact of income and price, even though multicollinearity prevents us from deriving precise short-run estimates.

A11.8 *Show how this model may be derived from the demand function and the adaptive expectations process.*

The adaptive expectations process may be rewritten:

$$i_{t+1}^e = \lambda i_t + (1 - \lambda) i_t^e.$$

Substituting this into (1), one obtains:

$$B_t = \beta_1 + \beta_2 \lambda i_t + \beta_2 (1 - \lambda) i_t^e + u_t.$$

We note that if we lag (1) by one time period:

$$B_{t-1} = \beta_1 + \beta_2 i_t^e + u_{t-1}.$$

Hence:

$$\beta_2 i_t^e = B_{t-1} - \beta_1 - u_{t-1}.$$

Substituting this into the second equation above, one has:

$$B_t = \beta_1 \lambda + \beta_2 \lambda i_t + (1 - \lambda) B_{t-1} + u_t - (1 - \lambda) u_{t-1}.$$

This is equation (3) in the question, with $\gamma_1 = \beta_1\lambda$, $\gamma_2 = \beta_2\lambda$, $\gamma_3 = 1 - \lambda$, and $v_t = u_t - (1 - \lambda)u_{t-1}$.

Explain why inconsistent estimates of the parameters will be obtained if equation (3) is fitted using ordinary least squares (OLS). (A mathematical proof is not required. Do not attempt to derive expressions for the bias.)

In equation (3), the regressor B_{t-1} is partly determined by u_{t-1}. The disturbance term v_t also has a component u_{t-1}. Hence the requirement that the regressors and the disturbance term be distributed independently of each other is violated. The violation will lead to inconsistent estimates because the regressor and the disturbance term are contemporaneously correlated.

Describe a method for fitting the model that would yield consistent estimates.

If the first equation in this exercise is true for time period $t + 1$, it is true for time period t:

$$i_t^e = \lambda i_{t-1} + (1 - \lambda)i_{t-1}^e.$$

Substituting into the second equation in (a), we now have:

$$B_t = \beta_1 + \beta_2\lambda i_t + \beta_2\lambda(1 - \lambda)i_{t-1} + (1 - \lambda)^2 i_{t-1}^e + u_t.$$

Continuing to lag and substitute, we have:

$$B_t = \beta_1 + \beta_2\lambda i_t + \beta_2\lambda(1 - \lambda)i_{t-1} + \cdots + \beta_2\lambda(1 - \lambda)^{s-1}i_{t-s+1} + (1 - \lambda)^s i_{t-s+1}^e + u_t.$$

For s large enough, $(1 - \lambda)^s$ will be so small that we can drop the unobservable term i_{t-s+1}^e with negligible omitted variable bias. The disturbance term is distributed independently of the regressors and hence we obtain consistent estimates of the parameters. The model should be fitted using a nonlinear estimation technique that takes account of the restrictions implicit in the specification.

Suppose that u_t were subject to the first-order autoregressive process:

$$u_t = \rho u_{t-1} + \varepsilon_t$$

where ε_t is not subject to autocorrelation. How would this affect your answer to the second part of this question?

v_t is now given by:

$$v_t = u_t - (1 - \lambda)u_{t-1} = \rho u_{t-1} + \varepsilon_t - (1 - \lambda)u_{t-1} = \varepsilon_t - (1 - \rho - \lambda)u_{t-1}.$$

Since ρ and λ may reasonably be assumed to lie between 0 and 1, it is possible that their sum is approximately equal to 1, in which case v_t is approximately equal to the innovation ϵ_t. If this is the case, there would be no violation of the regression assumption described in the second part of this question and one could use OLS to fit (3) after all.

Suppose that the true relationship was actually:

$$B_t = \beta_1 + \beta_2 i_t + u_t \qquad (1*)$$

with u_t not subject to autocorrelation, and the model is fitted by regressing B_t on i_t and B_{t-1}, as in equation (3), using OLS. How would this affect the regression results?

The estimators of the coefficients will be inefficient in that B_{t-1} is a redundant variable. The inclusion of B_{t-1} will also give rise to finite sample bias that would disappear in large samples.

How plausible do you think an adaptive expectations process is for modelling expectations in a bond market?

The adaptive expectations model is implausible since the expectations process would change as soon as those traders taking advantage of their knowledge of it started earning profits.

A11.9 The regression indicates that the short-run income, price, and population elasticities for expenditure on food are 0.14, −0.10, and −0.05, respectively, and that the speed of adjustment is $(1 - 0.73) = 0.27$. Dividing by 0.27, the long-run elasticities are 0.52, −0.37, and −0.19, respectively. The income and price elasticities seem plausible. The negative population elasticity makes no sense, but it is small and insignificant. The estimates of the short-run income and price elasticities are likewise not significant, but this is not surprising given that the point estimates are so small.

A11.10 The table gives the result of the specification with a lagged dependent variable for all the categories of expenditure.

	OLS logarithmic regression									
	LGDPI		*LGP*		*LGPOP*		*LGCAT*(−1)		Long-run effects	
	coef.	s.e.	coef.	s.e.	coef.	s.e.	coef.	s.e.	*DPI*	*P*
ADM	−0.38	0.18	−0.10	0.06	2.03	0.74	0.68	0.09	−1.18	−0.33
BOOK	−0.36	0.20	−0.21	0.22	2.07	0.74	0.75	0.12	−1.46	−1.05
BUSI	0.10	0.13	0.03	0.18	0.78	0.45	0.72	0.11	0.33	0.09
CLOT	0.44	0.10	−0.40	0.07	0.01	0.32	0.43	0.09	0.77	−0.70
DENT	0.71	0.18	−0.46	0.16	−0.13	0.51	0.47	0.13	1.34	−0.87
DOC	0.23	0.14	−0.11	0.10	0.21	0.35	0.78	0.10	1.04	−0.52
FLOW	0.20	0.24	−0.31	0.27	0.07	0.98	0.75	0.11	0.81	−1.25
FOOD	0.14	0.09	−0.10	0.06	−0.05	0.19	0.73	0.11	0.53	−0.35
FURN	0.07	0.22	−0.07	0.22	0.82	0.91	0.68	0.12	0.21	−0.23
GAS	0.10	0.17	−0.06	0.03	−0.13	0.45	0.76	0.08	0.42	−0.26
GASO	0.32	0.11	−0.10	0.02	−0.59	0.25	0.80	0.06	1.56	−0.47
HOUS	0.30	0.05	−0.09	0.04	−0.13	0.10	0.73	0.05	1.11	−0.32
LEGL	0.40	0.14	0.10	0.09	−0.90	0.36	0.68	0.09	1.23	0.30
MAGS	0.57	0.21	−0.48	0.37	−0.56	0.44	0.55	0.12	1.27	−1.08
MASS	−0.28	0.29	−0.23	0.11	1.08	0.89	0.75	0.12	−1.14	−0.93
OPHT	0.30	0.24	−0.28	0.33	−0.45	0.85	0.88	0.09	2.48	−2.25
RELG	0.34	0.09	−0.71	0.17	1.25	0.38	0.51	0.09	0.68	−1.44
TELE	0.15	0.14	0.00	0.12	0.68	0.37	0.81	0.12	0.77	0.02
TOB	0.12	0.14	−0.12	0.05	−0.31	0.43	0.71	0.11	0.43	−0.43
TOYS	0.31	0.11	−0.27	0.08	1.44	0.47	0.47	0.12	0.58	−0.51

A11.11 *In his classic study Distributed Lags and Investment Analysis (1954), Koyck investigated the relationship between investment in railcars and the volume of freight carried on the US railroads using data for the period 1884–1939. Assuming that the desired stock of railcars in year t depended on the volume of freight in year $t-1$ and year $t-2$ and a time trend, and assuming that investment in railcars was subject to a partial adjustment process, he fitted the following regression equation using OLS (standard errors and constant term not reported):*

$$\widehat{I}_t = 0.077F_{t-1} + 0.017F_{t-2} - 0.0033t - 0.110K_{t-1} \qquad R^2 = 0.85$$

where $I_t = K_t - K_{t-1}$ is investment in railcars in year t (thousands), K_t is the stock of railcars at the end of year t (thousands), and F_t is the volume of freight handled in year t (ton-miles).

Provide an interpretation of the equation and describe the dynamic process implied by it. (Note: It is best to substitute $K_t - K_{t-1}$ for I_t in the regression and treat it as a dynamic relationship determining K_t).

Given the information in the question, the model may be written:

$$\begin{aligned} K_t^* &= \beta_1 + \beta_2 F_{t-1} + \beta_3 F_{t-2} + \beta_4 t + u_t \\ K_t - K_{t-1} &= I_t = \lambda(K_t^* - K_{t-1}). \end{aligned}$$

Hence:

$$I_t = \lambda\beta_1 + \lambda\beta_2 F_{t-1} + \lambda\beta_3 F_{t-2} + \lambda\beta_4 t - \lambda K_{t-1} + \lambda u_t.$$

From the fitted equation:

$$\begin{aligned} \widehat{\lambda} &= 0.110 \\ \widehat{\beta}_2 &= \frac{0.077}{0.110} = 0.70 \\ \widehat{\beta}_3 &= \frac{0.017}{0.110} = 0.15 \\ \widehat{\beta}_4 &= \frac{-0.0033}{0.110} = -0.030. \end{aligned}$$

Hence the short-run effect of an increase of 1 million ton-miles of freight is to increase investment in railcars by 7,000 one year later and 1,500 two years later. It does not make much sense to talk of a short-run effect of a time trend.

In the long-run equilibrium, neglecting the effects of the disturbance term, K_t and K_t^* are both equal to the equilibrium value $\overline{K}$ and F_{t-1} and F_{t-2} are both equal to their equilibrium value $\overline{F}$. Hence, using the first equation:

$$\overline{K} = \beta_1 + (\beta_2 + \beta_3)\overline{F} + \beta_4 t.$$

Thus an increase of one million ton-miles of freight will increase the stock of railcars by 940 and the time trend will be responsible for a secular decline of 33 railcars per year.

A11.12 *One researcher asserts that consistent estimates will be obtained if (2) is fitted using OLS and (1) is fitted using IV, with Y_{t-1} as an instrument for X_t. Determine whether this is true.*

(2) may indeed be fitted using OLS. Strictly speaking, there may be an element of bias in finite samples because of noncontemporaneous correlation between v_t and future values of Y_{t-1}.

We could indeed use Y_{t-1} as an instrument for X_t in (1) because Y_{t-1} is a determinant of X_t but is not (contemporaneously) correlated with u_t.

The other researcher asserts that consistent estimates will be obtained if both (1) and (2) are fitted using OLS, and that the estimate of β_2 will be more efficient than that obtained using IV. Determine whether this is true.

This assertion is also correct. X_t is not correlated with u_t, and OLS estimators are more efficient than IV estimators when both are consistent. Strictly speaking, there may be an element of bias in finite samples because of noncontemporaneous correlation between u_t and future values of X_t.