Chapter 1

The Demand Side — Web Appendix

1.1 Investment

Investment refers to the purchase of goods which are expected to yield a stream of services over the future. Examples are firms buying machinery, governments building roads or individuals buying houses. Note that, conventionally household expenditure on long-lived items such as cars or fridges is defined as durable consumption though it has all the features of investment. In this chapter we shall focus on investment by firms, and specifically on firms investing to increase their capital stock.

Tobin's q theory of investment

Just as households choose consumption to maximize the present value of their expected utility, firms choose investment to maximize the present value of their expected profits V_t, such that

$$V_t = E_t \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} \Pi_{t+i}, \tag{1.1}$$

where Π is the profit the firm makes in any period and r is the real interest rate. To choose the level of investment that maximises the present value of their expected profits, firms will want to invest up until the point where the marginal benefits (MB) of investment are equal to the marginal costs (MC).

The simplest way of thinking about this problem is to take the case where a company invests 1 unit. We assume that it pays 1 for the investment immediately. We then need to work out the marginal benefits and costs to the company of this extra unit of investment. The marginal benefit is the present value of the resultant stream of profits (i.e. the additional profits the investment will allow the company to make over its lifetime, discounted to the present period).

The production function of a firm shows how much output the firm can produce from a given amount of inputs. If we denote the firm's output as y_t, then we can write the production function as $y_t = F(N_t, K_t)$, where N and K are the inputs the production — labour and capital. To find the revenue gained from 1 additional unit of investment we need to multiply the marginal productivity of capital f_K (i.e. the first derivative of the production function with respect to capital) by the price of output (P). This gives us Pf_K, which can be interpreted as the additional revenue the firm gains from the extra unit of investment. If the investment is made in period t, the company will receive this extra revenue in every future period. We now need to take account of interest payments and depreciation to calculate the marginal benefit of the investment.

We assume the output from the investment comes at the end of each period, as does the payment of interest r, which the company could have got from investing 1 elsewhere; and that the investment depreciates by δ each period, which is paid at the beginning of the following period; these assumptions imply that the marginal benefits (MB) are given by:

$$\begin{aligned} MB &= Pf_K\left(\frac{1}{1+r} + \frac{1-\delta}{(1+r)^2} + \frac{(1-\delta)^2}{(1+r)^3} + \ldots\right) \\ &= Pf_K\left(\frac{1}{1+r}\right)\left(1 + \frac{1-\delta}{1+r} + \frac{(1-\delta)^2}{(1+r)^2} + \ldots\right). \end{aligned} \tag{1.2}$$

The last term in equation 1.2 is a geometric series with a first term of 1 and a ratio of $\frac{1-\delta}{1+r}$, so its sum to infinity is $\left(\frac{1}{1-\frac{1-\delta}{1+r}}\right) = \left(\frac{1+r}{r-\delta}\right)$. This means equation 1.2 becomes

$$MB = \frac{Pf_K}{r+\delta}. \qquad \text{(marginal benefit of investment)}$$

The marginal benefits need to be equal to the marginal costs for the firm to maximise their lifetime profits. The marginal cost of the investment is 1 in this simple case. The firm has invested 1 unit, which could instead have been distributed as current profits to shareholders. Hence, equating marginal benefits and marginal costs of 1 extra unit of investment:

$$\frac{Pf_K}{\delta + r} = 1 = \frac{MB}{MC}. \qquad \text{(first order condition for the optimal capital stock)}$$

The first order condition for the optimal capital stock says that a company should invest until this condition holds. The numerator is the value of the marginal product of an extra unit of investment and is the marginal benefit of investing another unit. The denominator is often referred to as the user cost of capital. If you invest a unit today, δ is lost in depreciation and the remainder only generates profits tomorrow, which are worth r less per unit in the next period.

We will now define *marginal* q as

$$q = \frac{P f_K}{\delta + r}. \qquad \text{(marginal q)}$$

If $q > 1$, the marginal benefit of investment exceeds the marginal cost, so firms should invest to increase the capital stock until $q = 1$. If $q = 1$ the capital stock is optimal. If $q < 1$ the firms should reduce their capital stock. The assumption of diminishing marginal productivity of capital is required for this system to move towards the optimal level of investment. For $q > 1$, the firm is investing and the benefit of each additional unit of investment falls if the production function exhibits diminishing marginal productivity of capital. Hence they will invest until the marginal benefit of investment has fallen to a level where it equals the marginal cost of investment.

This is often referred to as Tobin's q, after James Tobin, the Nobel prize winning economist, and it tells us how investment depends on the four factors on the right hand side of the equation. The table below shows how increases in these variables affect q and consequently I :

Variable	Effect on q	Effect on I
r	$\downarrow$	$\downarrow$
δ	$\downarrow$	$\downarrow$
P	$\uparrow$	$\uparrow$
f_K	$\uparrow$	$\uparrow$

The marginal q equation is a condition for the optimal capital stock and the quantity of investment in any period will be whatever is needed to bring the capital stock to this optimal level.