Statistics III

Variance and significance testing

34

Answers to additional problems

34.1 Using eqn. (32.1), we calculate the mean mark for Class 2 as $\bar{x} = 54.9\%$. We cite this result to 3 significant figures.

34.2 Using eqn. (32.1), we calculate the mean mark for Class 1 as $\bar{x} = 45.1\%$. We cite this result to 3 significant figures.

34.3 There are 10 students in each class so $n = 10$. The number of degrees of freedom $DF = n - 1 = 9$ for each class.

34.4 The sample standard deviation is calculated using eqn. 34.3 or in *EXCEL*™ using STDEV.S(. . .).

This gives a value of $\sum_{i=1}^{N}(x_i - \bar{x}_1)^2 = 234.9$ and $s_1 = 5.11$.

34.5 Following the same approach as in Additional Problem 34.4, $\sum_{i=1}^{N}(x_i - \bar{x}_2)^2 = 224.9$ and $s_2 = 5.00$.

34.6 We start by stating the assumptions and hypotheses for this test.

- We assume that the sample data follow a normal distribution.
- We select the usual significance level of 5%.
- We note that the overall national average is lower than that of both Class 1 and 2. The exam must be taken by many other students, who found it even more difficult.

34.7 We start by stating the hypotheses for this test.

- We select the usual significance level of 5%.
- The null hypothesis is that the average mark for the students in Class 2 is the same as the average mark collated from all students taking the exam.
- The alternative hypothesis is that the average mark for the students in Class 2 is greater than the overall average.
- Since the alternative test is that the average mark is greater than the overall mark, we must calculate or look up t_{critical} in a one-tailed *t*-table for a 95% confidence level.

By looking up a one-tailed *t* table or by using the *Excel*™ formula '**ABS(T.INV(0.05, 9))**', we can calculate $t_{\text{critical}} = 1.83$.

34.8 We can use eqn. 34.5 to calculate $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{54.9 - 40}{5.00/\sqrt{10}} = 9.43$.

Since $|\,t\,| = 9.43 > t_{\text{critical}} = 1.83$, then the result is statistically significant. We can reject the null hypothesis. The higher average exam mark for the students in Class 2 is statistically significant. The students should all feel pleased with their results overall—it was clearly a challenging exam!

34.9 We start by stating the assumptions and hypotheses for this test.

- We assume that the sample data for each class follow a normal distribution.
- We assume that the two sets of data are independent. In other words, no student is in both Class 1 and 2.
- We assume the variance is approximately equal for each set of data and therefore we can use eqn. (34.6).

- We select the usual significance level of 5%.
- The null hypothesis is that the average mark for the students in Class 1 is the same as for the students in Class 2.
- The alternative hypothesis is that the average mark for the students in Class 1 is not the same as for the students in Class 2.
- Since the alternative test is that the average mark is not the same for the students in Class 1 and 2, and we aren't testing whether one class average is greater or less than the other, we must calculate or look up $t_{critical}$ in a two-tail t-table for a 95% confidence level.

Since each set of data has 10 values, the number of degrees of freedom is $DF = n_1 + n_2 - 2 = 18$.

By looking up a t table for a probability of 0.05 and 18 degrees of freedom or by using the *Excel*™ formula '**=ABS(T.INV.2T(0.05, 18))**', we can calculate $t_{critical} = 2.10$.

34.10 We have already found the mean values and the sample standard deviation for each exam.

- For Class 1, $n_1 = 10$, $\bar{x}_1 = 45.1\%$ and $s_1 = 5.11$.
- For Class 2, $n_1 = 10$, $\bar{x}_2 = 54.9\%$ and $s_2 = 5.00$.

The pooled standard deviation s_{pool} can be calculated from eqn. (35.7).

$$s_{pool} = \sqrt{\frac{(10-1)\,5.11^2 + (10-1)\,5.00^2}{10+10-2}} = 5.054$$

Using these values in eqn. (35.6), gives

$$t = \frac{45.1 - 54.9}{5.054 \times \sqrt{\frac{1}{10} + \frac{1}{10}}} = -4.34$$

Since $|\,t\,| > t_{critical}$ (4.34 > 2.10), then the result is statistically significant. We can reject the null hypothesis. The higher average exam mark for the students in Class 2 is statistically significant.

We don't know the reason for this. It could be that the students in Class 1 have not studied the same topics or had less time to practise before the exam. Hopefully the statistical data will encourage everyone to review why Class 2 has a higher average mark than Class 1.