Graphs II

The equation of a straight-line graph

28

Answers to additional problems

28.1 All we need to decide now is a suitable choice of algebraic symbol. For example, time (in days) can be t and the amount of material can be n. Accordingly, the equation becomes,

Equation of a line $\quad y = m \quad x + c$

Our data $\quad n = 400\ t\ + 312$

28.2 Prior knowledge is a constant (average per student) so $c = 18$ per cent.

The way the mark depends on attendance is effectively a rate—how often the student attends. Accordingly, $m = 1.5$ per cent per lecture.

We next decide suitable choices of algebraic symbol. Let the mark be M and the number of visits to the lectures can be A for attendance. Accordingly, the equation is,

Equation of a line $\quad y = m \ \ x + \ c$

Our data $\quad M = 1.5\,A + 18$

28.3 To reduce the equation, we divide through by the factor adjacent to y,

$$\frac{\cancel{12}y}{\cancel{12}} = \frac{4x}{12} - \frac{6}{12}$$

Cancelling yields, $y = ⅓x - ½$. We may prefer to write this in decimals instead, as $y = 0.33x - 0.5$.

28.4 The gradient is 4.9 so the incomplete equation is, $y = -4.9x + c$.
Inserting the values of x and y for the known point yields, $\quad 2 = (-4.9 \times -5) + c$.

Therefore, $2 = 24.5 + c$

so $\quad c = -22.5$

The equation of the straight line is, $y = -4.9x - 22.5$.

28.5 Inserting numbers, temperature voltage coefficient $= \dfrac{(1.433\text{ V} - 1.456)\text{V}}{(56 - 25)\,°\text{C}}$

so $\quad$ temperature voltage coefficient $= \dfrac{-0.023\text{V}}{31\,°\text{C}}$

and $\quad$ temperature voltage coefficient $= -7.42 \times 10^{-4}\text{V}\,°\text{C}^{-1}$.

28.6 The gradient is 4.9 so the incomplete equation is, $\quad y = -4.9 \quad x \quad + \quad c.$
Inserting the values of x and y for the known point yields, $\quad 2 = (-4.9 \quad \times \quad -5) \quad + \quad c.$

Therefore, $2 = 24.5 + c$

so $\quad c = -22.5$

The equation of the straight line is, $y = -4.9x - 22.5$.

28.7 The height of the hill goes up 1 metre for every 20 metres forward. We define the gradient by eqn. (28.2), so the numerator is 1 m and the denominator is 20 m.

The gradient m is $\frac{1\text{m}}{20\text{m}} = 0.05$.

The units on top and bottom are both m so they cancel.

28.8 **Strategy**

1. We calculate the value of the compound variable, $\varepsilon\, c\, \ell$.
2. Knowing the value of $\varepsilon \times c \times \ell$, we substitute for *Abs* and $\varepsilon\, c\, \ell$ and solve for k.

Solution

1. $\varepsilon\, c\, \ell = 30 \text{ mol}^{-1} \text{ dm}^3 \text{ cm}^{-1} \times 0.23 \text{ mol dm}^{-3} \times 1 \text{ cm} = 6.9$.
2. $0.8 = 6.9 + k$ so $k = -6.1$.

28.9 **Strategy**

1. Insert known data to obtain a gradient, m
2. Knowing the gradient, establish the constant c

Solution

1. Inserting pH and voltmeter reading to obtain m, $\frac{E_2 - E_1}{\text{pH}_2 - \text{pH}_1}$

$$\text{so} \quad m = \frac{300 - 400\text{mV}}{5.5 - 4.2} = \frac{-100\text{mV}}{1.3} = -76.9 \text{ mV per pH unit}$$

After dividing throughout by the units of mV, the equation is $E = -76.9 \text{ pH} + c$.

2. Inserting data (it does not matter which pair of E and pH we use provided they are related),

$400 \text{ mV} = -76.9 \text{ mV} \times 4.2 + c$,

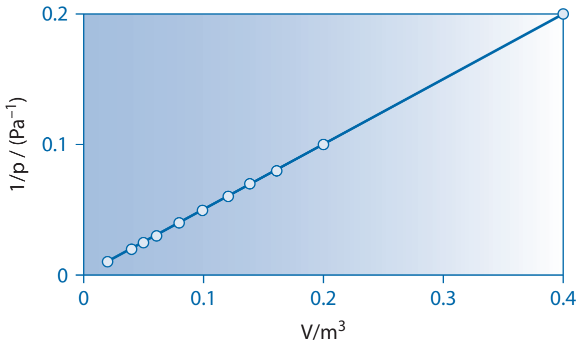
so $400 \text{ mV} = -322.98 \text{ mV} + c$, so $c = 722.98$ mV.

The equation of the straight line is, $E = 722.98 - 76.9$ pH.

28.10 We start by adding a third line to the table, for $1/p$,

V/m^3	0.01	0.02	0.025	0.03	0.04	0.05	0.06	0.07	0.08	0.1	0.2
p/Pa	50	25	20	16.5	12.5	10	8.3	7.2	6.2	5	2.5
$1/p$	0.02	0.04	0.05	0.06	0.08	0.1	0.12	0.14	0.16	0.2	0.4

We then draw a graph of $1/p$ (as y) against V (as x).



The gradient = 2 and the intercept is 0. The equation of the line is therefore, $1/p = 2 \times V$. The factor of 2 in the gradient tells us the number of moles of gas.