Integration I

Reversing the process of differentiation

19

Answers to additional problems

19.1 We first rewrite the equation in index form, $S = C_p \int T^{-1}\,\mathrm{d}T$
Equation (19.5) gives us the integral of this polynomial, so

$$S = C_p \ln T + c$$

where c is the constant of integration.

19.2 We imply a derivative when we talk about a 'gradient'. Stated otherwise, we start with an expression of the form $\frac{\mathrm{d}y}{\mathrm{d}x} = x^3$.
The original function must therefore have been $\frac{1}{4}x^4 + c$.

19.3 We again write the expression as an integration problem, $\Delta H = \int C_p\,\mathrm{d}T$
Substituting for C_p yields, $\Delta H = \int aT^3\,\mathrm{d}T$
We integrate, $\Delta H = \frac{1}{4}aT^4 + c$
where c is a constant of integration.

19.4 We start by rewriting the first term in index form as, $1.5/x^4 = 1.5\,x^{-4}$. We then recast the question into an integration problem, saying

$$y = \int 1.5x^{-4} + 5\sin 3x\,\mathrm{d}x$$

We can now integrate using eqns (19.1) and (19.3),

so $$y = \frac{1.5x^{-3}}{-3} - \frac{5}{3}\cos 3x + x$$

which we rewrite more tidily as $y = -\frac{1}{2x^3} - \frac{5}{3}\cos 3x + c$

19.5 We first rewrite in index form, $U = -Ar^{-6}$
We write as an integration problem, $\text{integral} = -A\int r^{-6}\,\mathrm{d}r$

so $$\text{integral} = -A \times \left(\frac{r^{-5}}{-5}\right) + c$$

where c is a constant of integration. The minus signs cancel. Then, with slight tidying, $\text{integral} = \frac{A}{5r^5} + c$

19.6 We write the integral as, $y = \int x^3 + x^2 - 1\,\mathrm{d}x$

then integrate as, $y = \frac{x^4}{4} + \frac{x^3}{3} - x + c$ where c is a constant of integration.

Inserting known data, $20 = \frac{2^4}{4} + \frac{2^3}{3} - 2 + c$

so $20 = 4 + \frac{8}{3} - 2 + c$

we calculate, $c=\dfrac{46}{3}$

to obtain, $y=\dfrac{x^4}{4}+\dfrac{x^3}{3}-x+\dfrac{46}{3}$

19.7 The term nRT is a constant so we write it outside the integration sign. We then write the appropriate integration symbolism, and say,

$$\int 1\,\mathrm{d}G = nRT\int \frac{1}{p}\,\mathrm{d}p$$

Integration with eqn. (19.5) gives,

$G = nRT \ln p + c$ where c is a constant of integration.

- The 1 on the left-hand side during the integration is a mathematical dodge that allows us to integrate, see p. 400.

19.8 We rewrite the concentration term in index form, $\int [\mathrm{A}]^{-2}\,\mathrm{d}[\mathrm{A}] = -k\int \mathrm{d}t$

then integrate, $-[\mathrm{A}]^{-1} = -kt + c$

where c is a constant of integration.

Multiplying throughout by –1 and tidying yields, $1/[\mathrm{A}] = k\,t - c$

19.9 Remember that $1/[\mathrm{A}]^3 = [\mathrm{A}]^{-3}$

Integrating yields, $-\tfrac{1}{2}[\mathrm{A}]^{-2} = -kt + c$

where c is a . Tidying and multiplying throughout by –1 yields, $\dfrac{1}{2[\mathrm{A}]^2} = kt - c$

19.10 **1.** The gradient, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{x} + \exp(2x)$.

We obtain the equation of the line by integration.
We first write the equation as an integration problem, $y = \int \dfrac{1}{x} + \exp(2x)\,\mathrm{d}x$
then integrate term by term,

$$y = \ln x + \tfrac{1}{2}\exp(2x) + c$$

where c is a constant of integration.

2. We then insert the known data,

$$202 = \ln 3 + \tfrac{1}{2}\exp(2 \times 3) + c$$

so $202 = 1.10 + (\tfrac{1}{2} \times 403.4) + c$

and $c = -0.8$

The full equation is, $y = \ln x + \tfrac{1}{2}\exp(2x) - 0.8$