Differentiation II

Differentiating other functions

14

Answers to additional problems

14.1 The box on p. 295 tells us the exponential can be written as the following series,

$$y = e^{2x} = 1 + \frac{(2x)^1}{1!} + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} \cdots$$

Differentiation yields,

$$\frac{dy}{dx} = \frac{2x^0}{1!} + \frac{2 \times 2^2 x^1}{2!} + \frac{3 \times 2^3 x^2}{3!} \cdots$$

Cancelling yields,

$$\frac{dy}{dx} = 2\left(1 + \frac{(2x)^1}{1!} + \frac{(2x)^2}{2!} \cdots\right) = 2\exp(2x)$$

14.2 Putting terms into eqn (14.2), $\frac{d[A]_t}{dt} = -k[A]_0 \exp(-kt)$

- The argument of the exponential remains unchanged following differentiation.
- The factor before the exponential operator has been multiplied by the derivative of the argument (in this example by $-k$).

14.3 We will start by multiplying the left-hand side of eqn. (1) by $dp \div dp$ (which is 1) and obtain,

$$\frac{d\ln p}{dT} \times \frac{dp}{dp} = \frac{d\ln p}{dp} \times \frac{dp}{dT}.$$

From eqn. (14.3), $\frac{d\ln p}{dp} = \frac{1}{p}$.

$$\frac{d\ln p}{dT} \times \frac{dp}{dp} = \frac{dp}{dT} \times \frac{1}{p}$$

so rewriting the left-hand side of eqn. (1) yields, $\frac{dp}{dT}\frac{1}{p} = \frac{\Delta H_{vap}}{RT^2}$.

Cross multiplying by p generates eqn. (2).

14.4 We can rewrite the argument of the logarithm as cT. Here c is a jumble of all the constants in the question. The expression becomes $S_m = R \ln cT$. From eqn. (14.3), the derivative of the logarithm's argument is $1/T$. So all the constants in the logarithm vanish.

There is also a constant written in front of the logarithm term, R. Equation (14.4) tells us to multiply it by $1/T$. The answer is therefore,

$$\frac{dS}{dT} = \frac{R}{T}$$

- The bracket vanishes because the source equation is a logarithm of the kind ln (ax) and eqn. (14.3) tells us the factor a vanishes. In this example the factor a is $(e^{5/2} kT/p^{\ominus} \Lambda^3)$.

14.5 For simplicity, we first rewrite the equation as $\psi = b\sin ax$, where a and b are constants. Using eqn. (14.6), we say, $a = \frac{n\pi}{L}$, $b = \frac{2}{L}$, and

$$\mathrm{d}\psi/\mathrm{d}x = (ab)\cos bx$$

Re-inserting for a and b yields, $\frac{\mathrm{d}\psi}{\mathrm{d}x} = \left(\frac{2n\pi}{L^2}\right)\cos\left(\frac{n\pi x}{L}\right)$.

- The argument of the trigonometric function has not altered during differentiation.

14.6 We differentiate, as $\frac{\mathrm{d}\psi}{\mathrm{d}x} = kA\cos(kx) - kB\sin(kx) = k\{A\cos(kx) - B\sin(kx)\}$

- As usual, the arguments of both the sine and cosine terms remain unchanged in this derivative.
- The factors before the sine and cosine operators have been multiplied by the derivative of the argument (in both cases here, by k). This is an example of the chain rule covered in more detail in Chapter 15.

The second differential, which we discuss in Chapter 17, is:

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} = kA \times -k\sin(kx) - kB \times k \quad \cos(kx)$$

which can be simplified further to give

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} = -k^2\psi$$

This form of wavefunction is a possible solution for the Schrödinger equation.

14.7 The domain of the argument means the volume V must always be positive. (In fact, a negative volume makes no physicochemical sense anyway.) The derivative of the logarithm is the reciprocal of the argument,

$$\frac{\mathrm{d}\Delta S}{\mathrm{d}V} = nR \times \frac{1}{V} = \frac{nR}{V}$$

We lose the c term during differentiation because it's a constant.

14.8 The derivative of the logarithm is the reciprocal of the argument,

$$\frac{\mathrm{d}\Delta S}{\mathrm{d}T} = C_V \times \frac{1}{T} = \frac{C_V}{T}$$

14.9 The power series is $\psi = j\left(1 + \frac{\left(-\frac{zr}{a_0}\right)^1}{1!} + \frac{\left(-\frac{zr}{a_0}\right)^2}{2!} + \frac{\left(-\frac{zr}{a_0}\right)^3}{3!} + \ldots\right)$

For simplicity, we say $j\left(1 + \frac{(kr)^1}{1!} + \frac{(kr)^2}{2!} + \frac{(kr)^3}{3!}\right)$, where $k = -\frac{z}{a_0}$

$$\frac{\mathrm{d}\psi}{\mathrm{d}r} = j\left(\frac{k}{1!} + \frac{2k^2r}{2!} + \frac{3k^3r^2}{3!} + \cdots\right) \text{ which has a common factor of } jk$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}r} = jk\left(1 + \frac{kr^1}{1!} + \frac{k^2r^2}{2!} + \cdots\right) = jk\mathrm{e}^{kr}$$

We again demonstrate the validity of eqn. (14.2).

14.10 Inserting terms into eqn. (14.2),

$$\frac{\mathrm{d}\psi}{\mathrm{d}r} = \left(\frac{\zeta^3}{\pi}\right)^{1/2} \times -\zeta \exp(-\zeta r)$$

The pre-exponential factor has not changed.

The coefficient from the differentiation

The exponential's argument always remains constant

To tidy the derivative, we note how $(\zeta^3)^{1/2}$ is $\zeta^{3/2}$. and $\pi^{1/2} = \sqrt{\pi}$. Therefore,

$$\frac{\mathrm{d}\psi}{\mathrm{d}r} = \left(\frac{\zeta^{3/2}}{\sqrt{\pi}}\right) \times \{-\zeta\} \times \exp(-\zeta r)$$

so $$\frac{\mathrm{d}\psi}{\mathrm{d}r} = \left(\frac{\zeta^{5/2}}{\sqrt{\pi}}\right) \exp(-\zeta r)$$