Differentiation I

Rates of change, tangents, and differentiation

13

Answers to additional problems

13.1 The equation can be re-written as, $v = \ell t^{-1}$.

so $\dfrac{dv}{dt} = -1 \times \ell \times t^{-2} = -\dfrac{\ell}{\tau^2}$

13.2 We start by rewriting the expression slightly, as $\tau = (2^{1/2}/\pi)\,(\Delta\nu)^{-1}$ where the first term in brackets is a constant.

Differentiating with eqn. (12.3) gives, $\dfrac{d\tau}{d(\nu\Delta)} = -1 \times (2^{1/2}/\pi)\,(\Delta\nu)^{-2}$

We will probably want to rewrite this result as, $\tau = \dfrac{2^{\frac{1}{2}}}{\pi(\Delta\nu)^2}$

13.3 Using eqn. (13.2), we say, $\dfrac{dI}{d\nu} = \frac{1}{2} \times k \times \nu^{-\frac{1}{2}} = \dfrac{k}{2\sqrt{\nu}}$

13.4 $\dfrac{d\mu}{dT} = \dfrac{3}{2} \times kT^{\frac{1}{2}}$

Tidying yields, $\dfrac{d\mu}{dT} = \dfrac{3k}{2}T^{\frac{1}{2}}$ or $\dfrac{3kT^{\frac{1}{2}}}{2}$ or even $\dfrac{3k\sqrt{T}}{2}$

13.5 $\dfrac{dM}{dc} = \dfrac{1}{n} \times kc^{\left(\frac{1}{n}-1\right)}$

It might be worth tidying this expression slightly as $\dfrac{dM}{dc} = \dfrac{k}{n}c^{\left(\frac{1}{n}-1\right)}$ or $\dfrac{kc^{\left(\frac{1}{n}-1\right)}}{n}$

13.6 The equation can be rewritten as $V = \left(\dfrac{\mu_1\mu_2}{4\pi\varepsilon_0}\right) \times \dfrac{1}{r^3}$ or $V = \left(\dfrac{\mu_1\mu_2}{4\pi\varepsilon_0}\right) \times r^{-3}$ where the bracket in each remains constant.

Therefore, $\dfrac{dV}{dr} = -3\left(\dfrac{\mu_1\mu_2}{4\pi\varepsilon_0}\right) \times r^{-4} = -\dfrac{3\mu_1\mu_2}{4\pi\varepsilon_0 r^4}$

13.7 We first rewrite this equation slightly, as,

$$V_{\text{eff}} = \left(-\frac{Ze^2}{4\pi\varepsilon_0}\right) \times r^{-1} + \left(\frac{l(l+1)\hbar^2}{2\mu}\right) r^{-2}$$

where both the bracketed terms are wholly constant.

$$\frac{dV_{\text{eff}}}{dr} = -1 \times \left(-\frac{Ze^2}{4\pi\varepsilon_0}\right) \times r^{-2} + -2 \times \left(\frac{l(l+1)\hbar^2}{2\mu}\right) r^{-3}$$

Tidying up yields, $\dfrac{dV_{\text{eff}}}{dr} = \dfrac{Ze^2}{4\pi\varepsilon_0 r^2} - \dfrac{2l(l+1)\hbar^2}{2\mu r^3}$

The two factors of 2 in the right-hand term cancel, leaving,

$$\frac{dV_{\text{eff}}}{dr} = \frac{Ze^2}{4\pi\varepsilon_0 r^2} - \frac{l(l+1)\hbar^2}{\mu r^3}$$

13.8 We can rewrite the equation slightly, as $I = \left(I_{\mathrm{o}} \dfrac{\pi\alpha^2}{\varepsilon_{\mathrm{r}}^2 r^2} \sin^2\phi \right) \lambda^{-4}$ where the bracketed term is constant.

$$\frac{\mathrm{d}I}{\mathrm{d}\lambda} = -4 \times \left(I_{\mathrm{o}} \frac{\pi\alpha^2}{\varepsilon_{\mathrm{r}}^2 r^2} \sin^2\phi \right) \lambda^{-5}$$

Tidying the derivative slightly yields, $\dfrac{\mathrm{d}I}{\mathrm{d}\lambda} = -I_{\mathrm{o}} \dfrac{4\pi\alpha^2}{\varepsilon_{\mathrm{r}}^2 r^2 \lambda^5} \sin^2\phi$

13.9 The equation can be rewritten as, $p = \dfrac{RT}{V_{\mathrm{m}}} + \dfrac{RTB}{V_{\mathrm{m}}^2} + \dfrac{RTC}{V_{\mathrm{m}}^3}$, and thence

$$p = (RT)\mathrm{V}_{\mathrm{m}}^{-1} + (RTB)V_{\mathrm{m}}^{-2} + (RTC)V_{\mathrm{m}}^{-3}$$

Each term contains a term of the form V_{m}^{-n}, therefore

$$\frac{\mathrm{d}p}{\mathrm{d}V_{\mathrm{m}}} = (-1) \times (RT)V_{\mathrm{m}}^{-2} + (-2) \times (RTB)V_{\mathrm{m}}^{-3} + (-3) \times (RTC)V_{\mathrm{m}}^{-4}$$

Tidying yields, $\dfrac{\mathrm{d}p}{\mathrm{d}V_{\mathrm{m}}} = -1 \times \left(\dfrac{RT}{V_{\mathrm{m}}^2} \right) - 2 \times \left(\dfrac{RTB}{V_{\mathrm{m}}^3} \right) - 3 \times \left(\dfrac{RTC}{V_{\mathrm{m}}^4} \right)$

Factorizing simplifies further, $\dfrac{\mathrm{d}p}{\mathrm{d}V_{\mathrm{m}}} = -RT \times \left[\left(\dfrac{1}{V_{\mathrm{m}}^2} \right) + \left(\dfrac{2B}{V_{\mathrm{m}}^3} \right) + \left(\dfrac{3C}{V_{\mathrm{m}}^4} \right) \right]$

13.10 The equation can be rewritten as, $b = \dfrac{qz^3\varepsilon F}{24\pi\varepsilon_0 R} \left(\dfrac{2}{\varepsilon R} \right)^{1/2} T^{-3/2}$

Therefore, $\dfrac{\mathrm{d}b}{\mathrm{d}T} = -\dfrac{3}{2} \times \dfrac{qz^3\varepsilon F}{24\pi\varepsilon_0 R} \left(\dfrac{2}{\varepsilon R} \right)^{1/2} T^{-5/2}$

We might choose to rewrite as $\dfrac{\mathrm{d}b}{\mathrm{d}T} = -\dfrac{qz^3 F}{16\pi\varepsilon_0} \left(\dfrac{2\varepsilon}{R^3 T^5} \right)^{1/2}$