Advanced BODMAS

Rearranging equations with more complicated functions

12

Answers to additional problems

12.1 The I term appears within the denominator of the fraction inside the bracket, which is inconvenient. Using the laws of logarithms, we can rewrite the equation slightly.

$$-\ln\left(\frac{I_0}{I}\right) = -c\,\ell\,\varepsilon \qquad \text{we multiply each side by } -1$$

$$\ln\left(\frac{I}{I_0}\right) = -c\,\ell\,\varepsilon$$

This follows because $\left(\frac{I_0}{I}\right) = \left(\frac{I}{I_0}\right)^{-1}$ and $\log a^n = n \log a$.

We now take the inverse function of the logarithm,

$$\left(\frac{I}{I_0}\right) = \exp(-c\,\ell\,\varepsilon)$$

Finally, we multiply both sides by I_0

$$I = I_0 \exp(-c\,\ell\,\varepsilon)$$

12.2 We first divide both sides by A in order to get the exponential on its own

$$\frac{k}{A} = \exp\left(-\frac{E_a}{RT}\right)$$

Then take the inverse function of the exponential, which is a natural logarithm ln

$$\ln\left(\frac{k}{A}\right) = -\frac{E_a}{RT}$$

We multiply both sides by $-RT$

$$-RT\ln\left(\frac{k}{A}\right) = E_a$$

Finally, we can remove the minus sign before the logarithm by inverting the logarithm ($\ln(a/b) = \ln(b/a)^{-1} = -\ln(b/a)$)

$$E_a = RT\ln\left(\frac{A}{k}\right)$$

12.3 **1.** We MULTIPLY both sides by 3, $3V = 4\pi r^3$

2. We DIVIDE both sides by 4π, $\frac{3V}{4\pi} = r^3$

3. We take the cube ROOT of both sides, $r = \sqrt[3]{\frac{3V}{4\pi}}$

4. Only now do we insert terms $r = \sqrt[3]{\frac{3 \times 5.56 \times 10^{-31}\,\text{m}^3}{4\pi}}$ so $r = 5.1 \times 10^{-11}$ m.

Don't forget to use brackets when dividing by 4π.

12.4 We first simplify the left-hand side so it comprises only one term, which includes $\Delta G^{\ominus}_{T_2}$

$$\frac{\Delta G^{\ominus}_{T_2}}{T_2} = \frac{\Delta G^{\ominus}_{T_1}}{T_1} + \Delta H^{\ominus}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

We then multiply both sides by T_2

$$\Delta G^{\ominus}_{T_2} = \frac{T_2 \Delta G^{\ominus}_{T_1}}{T_1} + T_2 \Delta H^{\ominus}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

The second term on the right-hand side could also be rewritten, so the equation becomes

$$\Delta G^{\ominus}_{T_2} = T_2\left(\frac{\Delta G^{\ominus}_{T_1}}{T_1} + \Delta H^{\ominus}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right) \quad \text{or} \quad \Delta G^{\ominus}_{T_2} = \frac{T_2 \Delta G^{\ominus}_{T_1}}{T_1} + \Delta H^{\ominus}\left(1 - \frac{T_2}{T_1}\right)$$

12.5 Looking at the right-hand side of the equation shows how the concentration [**I**] was first SQUARE ROOTED to form $\sqrt{[\mathbf{I}]}$ which we then MULTIPLIED by k [aromatic].

The first step when obtaining [(**I**)] is therefore to DIVIDE both sides of the equation by k [aromatic]. We will treat k [aromatic] as a compound variable. We therefore perform the inverse operation, and DIVIDE both sides in a single step,

$$\frac{\text{rate}}{k\,[\text{aromatic}]} = \sqrt{[\mathbf{I}]}$$

Since the right-hand side is a SQUARE ROOT to obtain [**I**] from $\sqrt{[\mathbf{I}]}$ we perform the inverse operation and SQUARE both sides,

$$\left(\frac{\text{rate}}{k\,[\text{aromatic}]}\right)^2 = \left(\sqrt{[\mathbf{I}]}\right)^2$$

The SQUARE of a SQUARE ROOT produces the thing itself. In this case, it produces [**I**],

$$[\mathbf{I}] = \left(\frac{\text{rate}}{k\,[\text{aromatic}]}\right)^2$$

12.6 We first make the logarithm the subject, so we subtract k and divide by $-a$,

$$\log_{10}[\text{F}^-] = \frac{emf - k}{-a}$$

We multiply both top and bottom of the right-hand side by –1 and then take the inverse function of $\log_{10}$, which is 10^x

$$[\text{F}^-] = 10^{\left(\frac{k - emf}{a}\right)}$$

12.7 To make c the subject of the equation, we first SUBTRACT $\Lambda°$ from both sides,

$$\Lambda - \Lambda° = -b\sqrt{c}$$

Note how the minus sign persists on the right-hand side. Next, we note how $\sqrt{c}$ has been MULTIPLIED by a factor of $-b$, so we DIVIDE both sides by $-b$,

$$\frac{\Lambda - \Lambda°}{-b} = \sqrt{c}$$

Finally we perform the inverse operation to SQUARE ROOT, and square both sides,

$$c = \left(\frac{\Lambda - \Lambda°}{-b}\right)^2$$

We could rewrite this result, multiplying top and bottom within the bracket by '–1',

$$c = \left(\frac{\Lambda^\circ - \Lambda}{b}\right)^2$$

- It might have been easier to rewrite the source equation as $\Lambda = \Lambda^\circ + (-b\sqrt{c})$ to make it clear which term we should isolate first.

12.8 We first need to simplify the right-hand side of the equation by separating the bracket from the remainder of the equation, getting it on its own. So we cross-multiply

$$-\frac{R}{\Delta H^{\ominus}}\ln\left(\frac{K_2}{K_1}\right) = \left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

The bracket on the right-hand side is now redundant

$$-\frac{R}{\Delta H^{\ominus}}\ln\left(\frac{K_2}{K_1}\right) = \frac{1}{T_2} - \frac{1}{T_1}$$

We then move the $1/T_1$ term

$$\frac{1}{T_2} = \frac{1}{T_1} - \frac{R}{\Delta H^{\ominus}}\ln\left(\frac{K_2}{K_1}\right)$$

Finally, we take the reciprocal of both sides,

$$T_2 = 1\Big/\left(\frac{1}{T_1} - \frac{R}{\Delta H^{\ominus}}\ln\left(\frac{K_2}{K_1}\right)\right) \quad \text{so} \quad T_2 = \left(\frac{1}{T_1} - \frac{R}{\Delta H^{\ominus}}\ln\left(\frac{K_2}{K_1}\right)\right)^{-1}$$

12.9 **1.** The sixth ROOT has been taken, to form $\sqrt[6]{\upsilon}$.

2. $\sqrt[6]{\upsilon}$ has been DIVIDED into a large collection of constants $0.62nFAc\,D^{2/3}\omega^{1/2}$. This collection certainly looks formidable, but we do not really need to think about them. If it makes the equation look less scary, rewrite it as $k \div \sqrt[6]{\upsilon}$, where k represents all these constants bundled together.

To rearrange the equation, we reverse these two operations,

1. To obtain $\sqrt[6]{\upsilon}$ on its own, we MULTIPLY both sides of the equation by $\sqrt[6]{\upsilon}$ and DIVIDE both sides by I. We obtain,

$$\sqrt[6]{\upsilon} = \frac{0.62\,nFA\,c\,D^{2/3}\omega^{1/2}}{I}$$

2. To reverse the sixth ROOT, we must take the sixth POWER. We obtain,

$$\left(\sqrt[6]{\upsilon}\right)^6 = \left(\frac{0.62\,nFA\,c\,D^{2/3}\omega^{1/2}}{I}\right)^6$$

The left-hand side then collapses to form

$$v = \left(\frac{0.62\,nFA\,c\,D^{2/3}\omega^{1/2}}{I}\right)^6$$

12.10 We first remove the pre-exponential factor by cross-multiplying, and combining the exponentials arguments using the first power law, eqn. (9.4)

$$\frac{kh}{Tk_B} = \exp\left(\frac{\Delta S^{\ddagger}}{R} + \left(-\frac{\Delta H^{\ddagger}}{RT}\right)\right)$$

We then take the inverse function of the exponential which is ln. Doing so deals with both terms on the right-hand side

$$\ln\left(\frac{kh}{Tk_B}\right) = \frac{\Delta S^{\ddagger}}{R} - \frac{\Delta H^{\ddagger}}{RT}$$

Before we go further, for historical reasons, it's usual to split the 'ln' term in a particular way, which we will show in 2 steps

$$\ln\left[\left(\frac{k}{T}\right) \times \left(\frac{k_B}{h}\right)^{-1}\right] = \frac{\Delta S^{\ddagger}}{R} - \frac{\Delta H^{\ddagger}}{RT}$$

$$\ln\left(\frac{k}{T}\right) - \ln\left(\frac{k_B}{h}\right) = \frac{\Delta S^{\ddagger}}{R} - \frac{\Delta H^{\ddagger}}{RT}$$

We next add the enthalpy term to both sides

$$\frac{\Delta S^{\ddagger}}{R} = \ln\frac{k}{T} - \ln\frac{k_B}{h} + \frac{\Delta H^{\ddagger}}{RT}$$

Finally, we multiply both sides by the gas constant

$$\Delta S^{\ddagger} = R\ln\frac{k}{T} - R\ln\frac{k_B}{h} + \frac{\Delta H^{\ddagger}}{T}$$