Powers II

Exponentials and logarithms

10

Answers to additional problems

10.1 The x in the expression $\log x$ relates to the power to which 10 must be raised to obtain a number.

$\log(10^{-5})$ is –5 and the pH of this solution is $-1 \times -5 = +5$.

10.2 We first take logs of both sides, $\log(\text{fraction remaining}) = \log(\tfrac{1}{2})^n$

Secondly, we simplify the term on the right-hand side by remembering from the third law of logarithms how $\log a^b = b \times \log a$.

Therefore, $\log(\text{fraction remaining}) = n \log(\tfrac{1}{2})$.

10.3 We first rearrange the expression slightly, making k the subject,

$$k = \frac{1}{t} \times \ln\left(\frac{[(\text{VIII})]_0}{[(\text{VIII})]_t}\right)$$

Secondly, we insert values for terms, $k = \dfrac{1}{1260\,\text{s}} \times \ln\left(\dfrac{0.01\ \text{mol dm}^{-3}}{8.09 \times 10^{-3}\ \text{mol dm}^{-3}}\right)$

so $k = \dfrac{1}{1260\,\text{s}} \times \ln(1.236) = \dfrac{1}{1260\,\text{s}} \times 0.212 = 1.68 \times 10^{-4}\ \text{s}^{-1}$.

10.4 The inverse function of ln is exponential, so we take the exponential of both sides,

$$\exp(\ln I) = \exp(a + b\eta)$$

The exponential of a logarithm of a term is the term itself so, $I = \exp(a + b\eta)$.

10.5 Using the second law of logarithms, $\ln k_2 - \ln k_1 = -\dfrac{E_a}{R}\left(\dfrac{1}{T_2} - \dfrac{1}{T_1}\right)$

10.6 The inverse function to $\log x$ is 10^x so we take 10^x of both sides,

so $10^{(\log \gamma)} = 10^{(-Az^+z^-\sqrt{I})}$

The inverse function of a function yields the thing itself. Accordingly, the left-hand side simplifies to,

$$\gamma = 10^{(-Az^+z^-\sqrt{I})}$$

10.7 We first split the two ln terms using the second laws of logarithms,

$$\ln k - \ln T = -\frac{\Delta H^{\ddagger}}{RT} + \frac{\Delta S^{\ddagger}}{R} + \ln k_B - \ln h$$

We then take the two ln terms from the right-hand side onto the left,

$$\ln k - \ln T - \ln k_B + \ln h = -\frac{\Delta H^{\ddagger}}{RT} + \frac{\Delta S^{\ddagger}}{R}$$

We then combine all the ln terms as a new single term on the left-hand side as,

$$\ln\left(\frac{kh}{Tk_B}\right) = -\frac{\Delta H^{\ddagger}}{RT} + \frac{\Delta S^{\ddagger}}{R}$$

10.8 First $E_{Cu^{2+},Cu} - E^{\ominus}_{Cu^{2+},Cu} = \frac{RT}{2F} \times \ln[Cu^{2+}]$

so $2F(E_{Cu^{2+},Cu} - E^{\ominus}_{Cu^{2+},Cu}) = RT \times \ln[Cu^{2+}]$

and $\frac{2F}{RT}(E_{Cu^{2+},Cu} - E^{\ominus}_{Cu^{2+},Cu}) = \ln[Cu^{2+}]$

It is common for some students to try to rearrange the equation as,

$$\ln[Cu^{2+}] = \frac{E_{Cu^{2+},Cu}}{E^{\ominus}_{Cu^{2+},Cu} + \frac{RT}{2F}}$$

This rearrangement is not correct. BODMAS will not allow it. The subtraction step must happen first.

We then take the inverse function of a logarithm (which is an exponential) of both sides,

$$\exp\left\{\frac{2F}{RT}(E_{Cu^{2+},Cu} - E^{\ominus}_{Cu^{2+},Cu})\right\} = \exp\left(\ln[Cu^{2+}]\right)$$

The exponential of a logarithm is the thing itself,

$$[Cu^{2+}] = \exp\left\{\frac{2F}{RT}(E_{Cu^{2+},Cu} - E^{\ominus}_{Cu^{2+},Cu})\right\}$$

We sometimes write this expression slightly differently, as

$$[Cu^{2+}] = \exp\left\{\frac{2F(E_{Cu^{2+},Cu} - E^{\ominus}_{Cu^{2+},Cu})}{RT}\right\}$$

10.9 First, using the second law of logarithms, we combine the two ln terms,

$$\ln\left(\frac{p_2}{p_1}\right) = -\frac{\Delta H^{\ominus}_{\text{vaporisation}}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

Secondly, we reverse the logarithm, by taking the exponential of both sides,

$$\left(\frac{p_2}{p_1}\right) = \exp\left[-\frac{\Delta H^{\ominus}_{\text{vaporisation}}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right]$$

The brackets on the left-hand side are now superfluous.

$$\frac{p_2}{p_1} = \exp\left[-\frac{\Delta H^{\ominus}_{\text{vaporisation}}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right]$$

We then multiply both sides of the equation by p_1,

$$p_2 = p_1 \exp\left[-\frac{\Delta H^{\ominus}_{\text{vaporisation}}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right]$$

10.10 First, we take the logarithm of *both* sides,

$$\ln m = \ln (k\, c^{1/n})$$

Secondly, using the first law of logarithms, we split the right-hand side,

$$\ln m = \ln k + \ln c^{1/n}$$

Thirdly, using the third law of logarithms, we simplify the final term,

Equation of a straight line $y = c + mx$

Linearized equation $\ln m = \ln k + \frac{1}{n} \ln c$

We have **linearized** an equation. Chapter 29 discusses that process in greater depth