Powers I

Introducing indices and powers

9

Answers to additional problems

9.1 In the equation $A = \ell^2$, the operation is a SQUARE on ℓ. The reverse operation is a SQUARE ROOT, so $\ell = \sqrt{A}$. Accordingly, $\ell = \sqrt{7.2} = 2.68$ cm.

9.2 Simple rearranging yields, $I = \dfrac{V}{R}$

We then insert the numbers, $I = \dfrac{100V}{10^{12}\Omega} = \dfrac{10^2 V}{10^{12}\Omega}$.
Using the second laws of powers, eqn. (9.5), we could then say, $I = 10^{(2-12)}$ A $= 10^{-10}$ A.

9.3 We first make the term $D^{2/3}$ the equation's subject, remembering that, $\dfrac{1}{a^{-1/n}} = a^{1/n}$

$$D^{2/3} = \frac{I\nu^{1/6}}{0.62\,nFAc_{\text{analyte}}\,\omega^{1/2}}$$

A power of ⅔ means the square of the third root. The inverse function takes the inverse power which is the 3/2 power,

$$\left(D^{2/3}\right)^{3/2} = \left(\frac{I\nu^{1/6}}{0.62\,nFAc_{\text{analyte}}\,\omega^{1/2}}\right)^{3/2}$$

The powers in the left-hand side vanish, as intended, to leave D,

$$\text{Accordingly, } D = \left(\frac{I\nu^{1/6}}{0.62\,nFAc_{\text{analyte}}\,\omega^{1/2}}\right)^{3/2}$$

Alternatively, we can divide keeping the negative index, $D^{2/3} = \left(\dfrac{I}{0.62\,nFAc_{\text{analyte}}\,\omega^{1/2}\nu^{-1/6}}\right)^{3/2}$

and $\left(D^{2/3}\right)^{3/2} = \left(\dfrac{I}{0.62\,nFAc_{\text{analyte}}\,\omega^{1/2}\nu^{-1/6}}\right)^{3/2}$ leading to, $D = \left(\dfrac{I}{0.62\,nFAc_{\text{analyte}}\,\omega^{1/2}\nu^{-1/6}}\right)^{3/2}$

9.4 We start with the equation, $I = 0.62\,nFA\,c_{\text{analyte}}\,D^{2/3}\omega^{1/2}\,\upsilon^{-1/6}$ from Additional Problem 9.3. We recall how a power of ½ is the same as a square root, and ⅙ is a sixth root. D raised to a power of ⅔ is the same as the third root of D^2. All terms written with no power in fact have a power of 1, so no root sign is needed.

We can therefore write,

$$I = \left(\frac{0.62\,nFAc_{\text{analyte}}\sqrt[3]{D^2}\,\sqrt{\omega}}{\sqrt[6]{\nu}}\right)$$

9.5 Firstly, the powers of 1 are a nonsense. We rewrite as $E = hc\,\lambda^{-1}$.

Next, λ^{-1} is the same as $1/\lambda^{+1} = 1/\lambda$.

So $E = hc \times 1/\lambda = \dfrac{hc}{\lambda}$

9.6 The unit in the numerator (top line) will be simply mol^1 or just mol. The IUPAC unit of length is the metre m, so the IUPAC unit of volume is m^3. Accordingly, the denominator (bottom line) is $(\text{m}^3)^{-1} = \text{m}^{-3}$. So the IUPAC unit of concentration is mol m^{-3}.

9.7 Multiplying out the bracket yields, energy $= 4\varepsilon\left(\dfrac{1}{r}\right)^{12} - 4\varepsilon\left(\dfrac{1}{r}\right)^{6}$

Rewriting in terms of indices, we obtain, $4\,\varepsilon(r^{-1})^{12} - 4\,\varepsilon\,(r^{-1})^{6} = 4\,\varepsilon\,r^{-12} - 4\,\varepsilon\,r^{-6}$

- The version of the Lennard–Jones equation here has been simplified for this problem.

9.8 First, we introduce the indices, $\omega = (2\pi\,c)^{-1}\left(\dfrac{k}{\mu}\right)^{\frac{1}{2}}$

Second, we split the bracketed term, $\omega = 2^{-1}\,\pi^{-1}\,c^{-1}\,k^{½}\,(1/\mu^{½})$

Thirdly, we simplify the final term, writing it in terms of indices, $\omega = 2^{-1}\,\pi^{-1}\,c^{-1}\,k^{½}\,\mu^{-½}$

In words, the compound unit '$\text{cm}^3\,\text{dm}^{-3}$' is 'cubic centimetres per cubic decimetre.'

9.9 $V = 100\ \text{dm}^3 = 100\ \text{dm}^3 \times 1\,000\ \text{cm}^3\ \text{dm}^{-3}$

Cancelling units yields, $V = 100\ \text{dm}^3 = 100 \times 1\,000\ \text{cm}^3$

Therefore, $V = 10^2 \times 10^3\ \text{cm}^3 = 10^{(2+3)}\ \text{cm}^3 = 10^5\ \text{cm}^3$

9.10 First, we introduce the indices, $\phi = (z^+\,z^-)^1\,(4\pi\,\varepsilon_0\varepsilon\,r^2)^{-1}$. There is no need to write a term with an index of 1 so we rewrite slightly as, $\phi = z^+z^-\,(4\pi\,\varepsilon_0\varepsilon\,r^2)^{-1}$

Secondly, we split the terms, $\phi = 4^{-1}\,z^+\,z^-\,\pi^{-1}\,\varepsilon_0^{-1}\,\varepsilon^{-1}\,r^{-2}$

- This slight rearrangement accommodates the convention that it's usual to place numbers at the beginning of an expression.