Algebra VII

Solving simultaneous linear equations

8

Answers to additional problems

8.1 The gradient of line (1) is 4 and the gradient of line (2) is 3. These gradients clearly differ, so the two lines are not parallel.

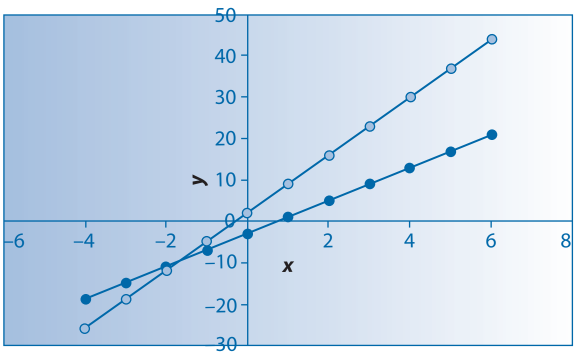
8.2 Normally we first reduce both equations and divide by the factor in front of y. The gradient is therefore 9.2/7.1. The intercepts differ, however, so the two lines are parallel but not the same.

In this case there is no need to reduce the equation since the factor against y is 7.1 in both equations and the factor before x in both equations is also the same at 9.2. Nevertheless, even without reduction, the gradients are the same which tells us these two lines are parallel.

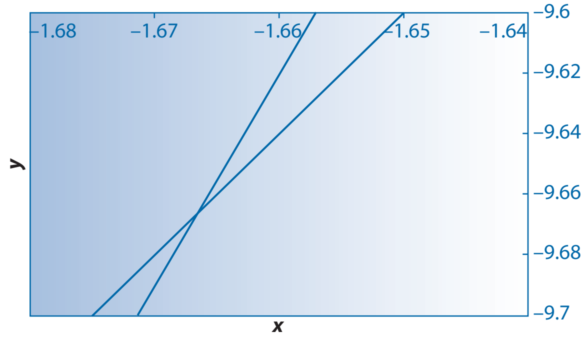
8.3 We first reduce equation (2) by dividing throughout by 2, because 2 is the factor against y. Equation (2) becomes, $y = 5x$. The gradients of the two lines are therefore the same, meaning the two lines are parallel. The intercept is different, though, so they are not the same line.

8.4 We first reduce the two equations, dividing by the respective factors against y. Equation (1) becomes $y = 5x + 55$, and equation (2) becomes, $y = 5x + 0.5$. The gradients of the two lines are therefore the same, indicating that the two lines are parallel. The intercept is different, so they are not the same line.

8.5 We would first draw the graph covering a wide data range. Such a graph suggests the point is intersection is probably, about, (–1.5, –9.5).



Secondly, we re-draw the graph, successively honing toward the values of x and y until we obtain the point of intersection.



In this way, we see how the point of intersection is (–1.667, –9.667).

- The 'honing' process is easier if we can use a spreadsheet program such as *Excel*™.

8.6 The equation for the first experiment is, $12 = m_{\text{sample}} + m_{\text{boat}}$ (1)
The equation for the second experiment is, $23 = 2m_{\text{sample}} + m_{\text{boat}}$ (2)

Subtracting eqn. (1) from eqn. (2) yields,

$$23 = 2m_{\text{sample}} + m_{\text{boat}} \quad (2)$$
$$12 = m_{\text{sample}} + m_{\text{boat}} \quad (1)$$
$$11 = m_{\text{sample}} \quad (2) - (1)$$

If $m_{\text{sample}} = 11$ g, re-inserting this number into either equation yields the result that the boat has a mass of 1 g.

We could have achieved the same result by multiplying eqn. (1) by 2 and then subtracting eqn. (1),

Subtracting eqn. (1) from eqn. (2) yields,

$$24 = 2m_{\text{sample}} + 2 \times m_{\text{boat}} \quad 2 \times (1)$$
$$23 = 2m_{\text{sample}} + m_{\text{boat}} \quad (2)$$
$$1 = m_{\text{boat}} \quad 2 \times (1) - (2)$$

so the boat weighs 1 g.

8.7 For the first solution, $0.0100 = \Lambda° - b\sqrt{1.100 \times 10^{-4}}$ (1)
For the second solution, $0.0200 = \Lambda° - b\sqrt{1.580 \times 10^{-3}}$ (2)

Calculating the roots yields,

$$0.0100 = \Lambda° - b \times 0.0105 \quad (1)$$
$$0.0200 = \Lambda° - b \times 0.0397 \quad (2)$$

We subtract eqn. (1) from eqn. (2),

$$0.0200 = \Lambda° - b \times 0.0397 \quad (2)$$
$$- \quad 0.0100 = \Lambda° - b \times 0.0105 \quad (1)$$
$$0.0100 = \quad - b \times 0.0292 \quad (2) - (1)$$

Dividing both sides by –0.0292 yields the result, $b = \frac{0.0100}{0.0292} = -0.342$

Knowing *b*, we can calculate Λ°. For example, using eqn. (1),

$$0.010 = \Lambda° - (-0.342 \times 0.0105)$$

so $0.0100 = \Lambda° + 0.00359,$

and $\Lambda° = 0.006\,41 \text{ S m}^{-1}$

8.8 Rearranging the second equation yields, $U = H - pV$

$$G = H - TS \qquad (1)$$
$$U = H - pV \qquad (2)$$

Subtracting eqn. (2) from eqn. (1) yields,

$$\begin{array}{rll} & G = H - TS & (1) \\ - & U = H - pV & (2) \\ \hline & G - U = -TS - (-pV) & (2) - (1) \\ \text{so} & G - U = pV - TS & \end{array}$$

We might re-state this as, $G = U + pV - TS$

We could have obtained the same result by substituting for H in eqn. (1),

$G = H - TS \quad \rightarrow \quad G = (U + pV) - TS$, which is the same result.

8.9 For the first solution, $\quad 0.276 = K - 0.059 \times 7.0 \quad (1)$
For the second solution, $\quad 0.502 = K - 0.059 \times x \quad (2)$

eqn. (2) – eqn. (1),

$$\begin{array}{rll} & 0.502 = K - 0.059 \times x & (2) \\ - & 0.276 = K - 0.059 \times 7.0 & (1) \\ \hline & 0.226 = -0.059\,(x - 7.0) & (2) - (1) \end{array}$$

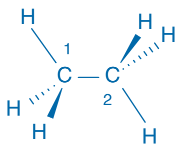
Dividing each side by –0.059, $-\dfrac{0.226}{0.059} = -3.8 = (x - 7.0)$

Therefore, $x = -3.8 + 7.0 = 3.2$.

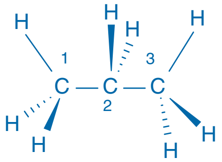
The pH of the unknown solution is 3.2.

We could have performed the same calculation by first using eqn. (1) to obtain K then using this value of K in eqn. (2).

8.10 It will probably help to draw the structures,



I



II

Ethane (**I**) has 1 C–C bonds and 6 C–H bonds.

Propane (**II**) has 2 C–C bonds and 8 C–H bonds.

If the enthalpy of a C–C bonds is $\Delta H_{\text{C–C}}$ and the enthalpy of a C–H bond is $\Delta H_{\text{C–H}}$, then we say,

For ethane (**I**), $\quad 1\,560 = \Delta H_{\text{C–C}} + 6\,\Delta H_{\text{C–H}} \quad (1)$

For propane (**II**), $\quad 2\,220 = 2\Delta H_{\text{C–C}} + 8\,\Delta H_{\text{C–H}} \quad (2)$

Subtracting (2) from 2 × (1) yields,

$$\begin{array}{rlcll} & 3\,120 = 2\Delta H_{\text{C–C}} & + & 12\,\Delta H_{\text{C–H}} & 2 \times (1) \\ - & 2\,220 = 2\Delta H_{\text{C–C}} & + & 8\,\Delta H_{\text{C–H}} & (2) \\ \hline & 900 = 4\,\Delta H_{\text{C–H}} & & & \end{array}$$

Therefore, $\Delta H_{\text{C–H}} = \frac{1}{4}$ of 900 kJ mol^{-1} = 225 kJ mol^{-1}.

Back substitution yields $\Delta H_{\text{C–C}} = 210$ kJ mol^{-1}.