Algebra I

Introducing notation, symbols, and operators

2

Answers to additional problems

2.1

$$\Delta y = y_{\text{final}} - y_{\text{initial}}$$
$$\Delta y = 32 - 12$$
$$\Delta y = 20\ \text{cm}$$

2.2

$$\Delta S = S_{\text{final}} - S_{\text{initial}}$$
$$\Delta S = (-32.5) - (-15.1)\ \text{J K}^{-1}\ \text{mol}^{-1}$$
$$\Delta S = -17.4\ \text{J K}^{-1}\ \text{mol}^{-1}$$

2.3 Inserting values into the equation above,

$$\Delta H_c^{\ominus} = \{(-393.51) + (2\times -285.83)\} - \{(1\times -74.81) + (2\times 0)\}$$
$$\Delta H_c^{\ominus} = (-965.17) - (-74.81)$$
$$\Delta H_c^{\ominus} = -890.36\ \text{kJ mol}^{-1}$$

O_2 is an element, so we **define** its value of $\Delta H_f^{\ominus}$ as zero. All the other values of $\Delta H_f^{\ominus}$ are negative, so they form exothermically. Respective values can be found in standard tables.

2.4

$$\sum_{i=2}^{5} i^2 = 2^2 + 3^2 + 4^2 + 5^2$$
$$\sum_{i=2}^{5} i^2 = 4 + 9 + 16 + 25 = 54$$

2.5

$$\sum_{i=4}^{12} 3i^2 = 3\times\left(4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + 11^2 + 12^2\right)$$
$$\sum_{i=4}^{12} 3i^2 = 3\times(16 + 25 + 36 + 49 + 64 + 81 + 100 + 121 + 144)$$
$$\sum_{i=4}^{12} 3i^2 = 3\times(636) = 1908.$$

2.6 Alkanes have the formula, C_nH_{2n+2}

There are 2 atom types C and H, therefore $i = 2$.

$$\text{Mass} = \sum_{i=1}^{2} N_i M_i$$
$$= N_C M_C + N_H M_H$$
$$= nM_C + (2n+2) M_H$$
$$= 12n + (2n+2)$$

2.7 A complete summation from $i = 1$ through to $i = \infty$ is clearly impossible, so we will work out values from $i = 1$ through to $i = 70$, and then look critically at the data.

i	1	2	3	4	5	6	7	8	9	10
$2/i^2$	2.0000	0.5000	0.2222	0.1250	0.0800	0.0556	0.0408	0.0313	0.0247	0.0200
i	11	12	13	14	15	16	17	18	19	20
$2/i^2$	0.0165	0.0139	0.0118	0.0102	0.0089	0.0078	0.0069	0.0062	0.0055	0.0050
i	21	22	23	24	25	26	27	28	29	30
$2/i^2$	0.0045	0.0041	0.0038	0.0035	0.0032	0.0030	0.0027	0.0026	0.0024	0.0022
i	31	32	33	34	35	36	37	38	39	40
$2/i^2$	0.0021	0.0020	0.0018	0.0017	0.0016	0.0015	0.0015	0.0014	0.0013	0.0013
i	41	42	43	44	45	46	47	48	49	50
$2/i^2$	0.0012	0.0011	0.0011	0.0010	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008
i	51	52	53	54	55	56	57	58	59	60
$2/i^2$	0.0008	0.0007	0.0007	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006
i	61	62	63	64	65	66	67	68	69	70
$2/i^2$	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004

The data in the table show that values of $2/i^2$ do converge, but not at all quickly.

The sum from $i = 1$ to $i = 10 = 3.099\,6$

The sum from $i = 1$ to $i = 20 = 3.192\,3$

The sum from $i = 1$ to $i = 30 = 3.224\,3$

The sum from $i = 1$ to $i = 40 = 3.240\,5$

The sum from $i = 1$ to $i = 50 = 3.250\,3$

The sum from $i = 1$ to $i = 60 = 3.256\,8$

The sum from $i = 1$ to $i = 70 = 3.261\,5$

For true convergence to n decimal places, the sum needs to be consistent to $(n+2)$ d.p. for two steps in a row.

Therefore, if we round the answer to 2 d.p., extending the summation process from $i = 60$ to $i = 70$ has not increased the value of the answer. Therefore, to 2 d.p. $\sum_{i=1}^{\infty} \frac{2}{i^2} = 3.26$.

2.8 $\prod(1.2, 4, 5.5, 7, 9) = 1.2 \times 4 \times 5.5 \times 7 \times 9 = 1663.2$

2.9

Care: Remember that each term is divided by '3', so the final denominator is '3 × 3 × 3' so 3^3.

$$\prod_{i=3}^{5} \frac{i^2}{3} = \frac{3^2}{3} \times \frac{4^2}{3} \times \frac{5^2}{3}$$

$$\prod_{i=3}^{5} \frac{i^2}{3} = \frac{9}{3} \times \frac{16}{3} \times \frac{25}{3}$$

$$\prod_{i=3}^{5} \frac{i^2}{3} = \frac{3600}{3^3}$$

$$\prod_{i=3}^{5} \frac{i^2}{3} = \frac{3600}{27} = 133.33$$

2.10 Rewriting the equation above to accommodate the problem,

$$K = \frac{[\mathrm{Cr^{IV}}] \times [\mathrm{Co^{III}}]^2}{[\mathrm{Cr^{VI}}] \times [\mathrm{Co^{II}}]^2}$$

Inserting values into this equation yields,

$$K = \frac{[0.09] \times [0.4361]^2}{[3.22 \times 10^{-9}] \times [2.9 \times 10^{-8}]^2}$$

so $$K = \frac{0.0171}{2.708 \times 10^{-24}}$$

and $K = 6.3 \times 10^{21}$.

Note how we express the answer to 2 s.f. because the initial data were given to 2 s.f.