Snowball Performance Final Analysis

Groups in Dynamics 2340 were tasked with designing and building snowball throwing devices by using methods and analysis techniques learned in class. These devices then competed in a "snowball fight" which consisted of three separate competitions involving distance, accuracy, and fire rate. Below is the explanation, results, and analysis of the design along with suggested improvements, recommendations, and what was learned from this project.

Design

The design my group decided to build was a device created by B. Jamieson. The device is a hand-held rifle-shaped catapult that moves in a circular motion. The force that throws the snowball comes from an elastic that is hooked to the bottom end of the leaver arm as shown in Image 1.1 (See Appendix A and B for more sketches and photos).

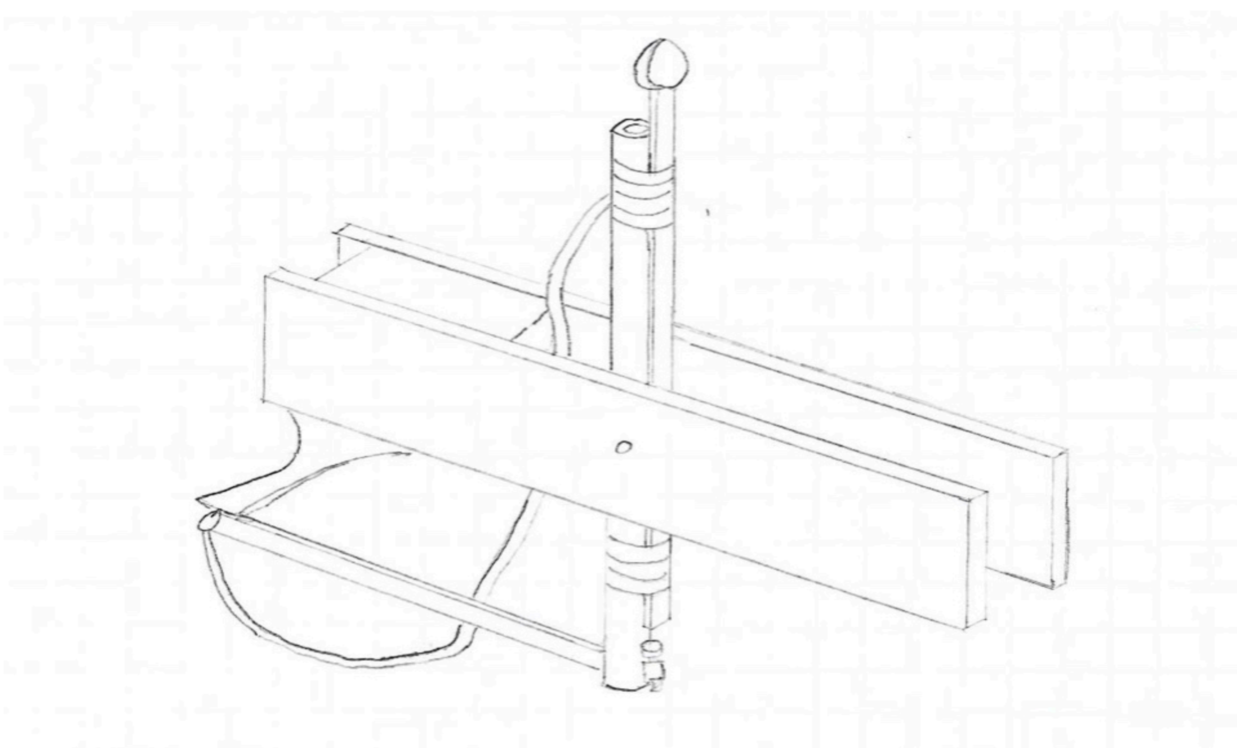


Image 1.1: A sketch of the device built.

To fire this device, the lever arm is pulled back using the rope attached from the top of the lever arm to the bottom of the grip. A snowball is then placed in the snowball holder at the top of the lever arm. To fire, the user releases the rope. This allows the elastic tourniquets to pull the bottom of the lever arm back, and the top of the lever arm to rotate forward. This in turn launches the snowball.

Results (Pre-Competition)

The initial design of the rifle consisted of five elastic tourniquets giving the force that propelled the top of the lever arm and in turn the snowball. Testing this prototype, the distance it could throw a snowball was twenty-one feet in length. Due to the desire to increase the distance, 5 more elastic tourniquets were added to make the total number of tourniquets ten. Testing this final design, the distance the snowball was thrown was 31 feet. Doing an analysis of the two designs from videos taken of the launch sequence gives data that explains this. As shown from the Image 1.2, focusing on first four points of the graph (where the snowball is still in the launcher) the acceleration of the snowball before it left the launcher of the final design was larger than the prototype.

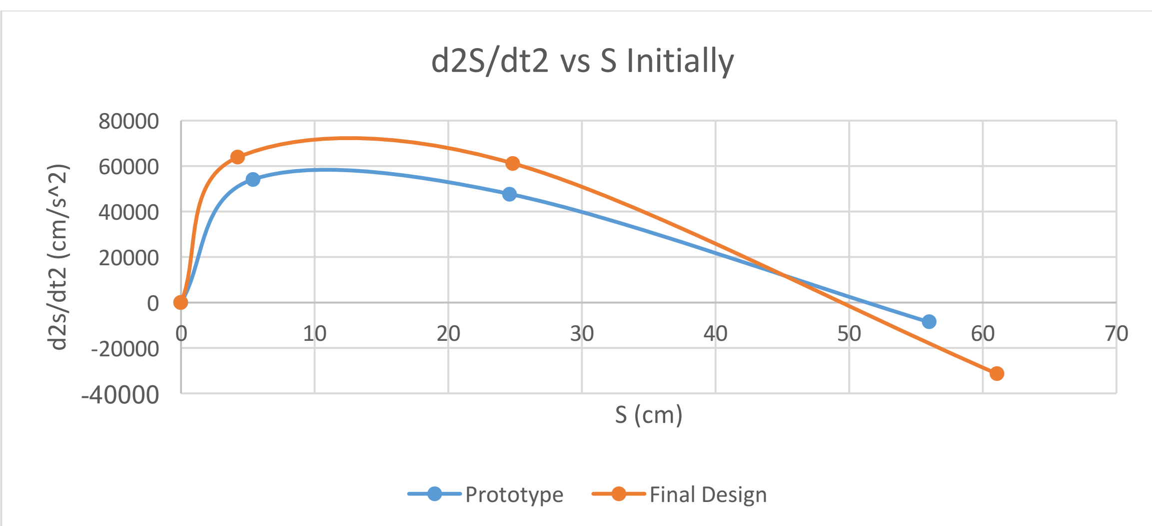


Image 1.2: A graph comparing d2S/dt2 vs the initial S values.

The final design reached a higher acceleration in a shorter distance compared to the prototype. The final design also made the snowball maintained a higher acceleration over the span of the path while it is still in the snowball holder. Looking at the additional graphs in the Excel file attached, it is demonstrated that the final design causes the lever arm and snowball to reach a higher speed in a shorter amount of time compared to the prototype.

Results (Competition)

Fire Rate

The goal of the fire rate competition was to fire as many snowballs in ten seconds as possible. Going into this competition, my group was confident that we would win. Implemented into the design were removable elastic tourniquets could be taken out of the bottom of the lever arm, so that the lever arm could be powered manually by the user. Doing this caused the lever arm to have a quick release and reload time. With S. holding a platter of snowballs and myself quickly placing them on the launcher, B. was able to fire ten snowballs quicker than the other three teams.

Accuracy

The goal of the accuracy competition was to fire three snowballs against a wall and have the snowballs hit within a close proximity. Our device fared well, making it through the first round but was eliminated by a stationary catapult in the second round. This competition was dominated by catapults, trebuchets and other ground based devices. This was due to the fact that the path of the snowballs in the larger catapults and trebuchets are almost the exact same every time a snowball was thrown. The sturdiness, balance, and the fix lever arm length of those designs cause this to happen. This was proven by the design that won this competition. The winning design had four legs spread apart with a fixed lever arm, making the design very sturdy, balanced and therefore accurate. Comparing the ground base devices to our design, it was evident that the path of the lever arm was not exacting the same. This is because there was movement of the arm holding the device when it is being fired. This changed the path each time the device was fired and therefore changed where the snowball hit the wall.

Distance

The objective of the distance competition was to determine which device could throw a snowball the farthest. This competition was not expected to be won. The force that the elastic tourniquets provided was not large enough to beat all of the ground based and man powered devices. Even if the elastics were able to provide double the force that they did in the final design, the distance the device could throw a snowball would still not be large enough to beat the winning design in this competition. In order to beat the winning design, not only would the force provided by the elastics have to be a lot greater, but the length of the lever arm would need to be increased as well. More on this in the improvements section.

Analysis

Below are calculations of the velocity and acceleration of the snowball when the ball is at the initial point, and the final point of launch.

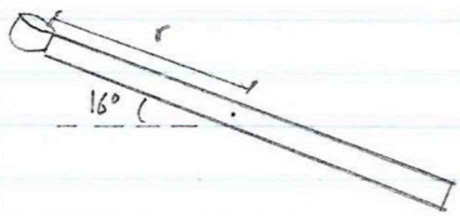
1) Velocities

To calculate the velocity of the snowball while it was under the force of the launcher, the arc length of the top of the snowball holder was calculated. This value was then used as the “S” value in the integral of momentum to determine the velocity at the point of launch. For the starting of the launch, the “S” value was assumed to be 0.0424m. This value was the S value at the second point of launch (0.033 seconds into the launch).

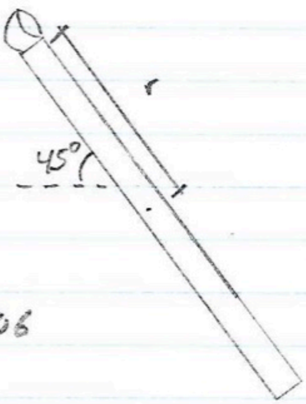
Path

FBD

1



2



$\Delta\theta = 45° - 16° = \underline{29°} \rightarrow \frac{29°}{360°} = 0.0806$

$r = 25cm = 0.25m$

$\rightarrow$ Circumference $= 2\pi r = 2\pi \cdot 0.25m = 1.57m$

$\therefore$ Path $= 0.0806 \cdot 1.57m = 0.1266\,m = \boxed{12.66\text{ cm}}$

Angular Velocity

$\int_{S_0}^{S_1} M\,ds = \frac{1}{2} I w^2 \rightarrow M(S_1) - M(S_0)^{0} = \frac{1}{2} I w^2$

$\therefore M(S) = \frac{1}{2} I w^2 \rightarrow \boxed{w^2 = \frac{2M(S)}{I}}$

$\rightarrow$ S value when at the start of launch will be 0.0424m (from excel file)

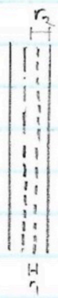
$\rightarrow$ S value when launcher is at launch point will be path length, 0.1266m.

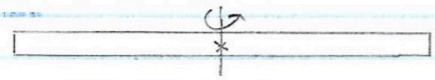
Solving the momentum equation for the angular velocity, the moment of inertia was for the lever arm was then calculated.

Solving for I values...

$$I = I_{arm} + I_{holder} + I_{snowball}$$

$I_{arm} + I_{holder}$ (assuming one-piece)





$$I_y = \frac{1}{12}\left(m\left(3\left(r_2^2 + r_1^2\right) + 0.5m^2\right)\right)$$

$$\therefore I_y = \frac{1}{12}\left(0.36\,kg\left(3\left(0.017^2 m + 0.013^2 m\right) + 0.5^2 m\right)\right)$$

$$= \frac{1}{12}\left(0.09049\,kg\cdot m^2\right) = 0.00754 = \underline{0.0075\,kg\cdot m^2}$$

$I_{snowball}$

$m = 150g = 0.150\,kg$ $\qquad I = I_b + I_{pt}$

$r_1 = 4cm = 0.04m$

$$I = \frac{2}{5} m r_1^2 + m r_2^2$$

$$= \frac{2}{5}(0.150\,kg)(0.04m)^2 + 0.150\,kg \cdot (0.25m)^2$$

$$= 0.000096\,kg\cdot m^2 + 0.009375\,kg\cdot m^2 = \underline{0.009471\,kg\cdot m^2}$$

$$\therefore I = 0.0075\,kg\cdot m^2 + 0.009471\,kg\cdot m^2$$

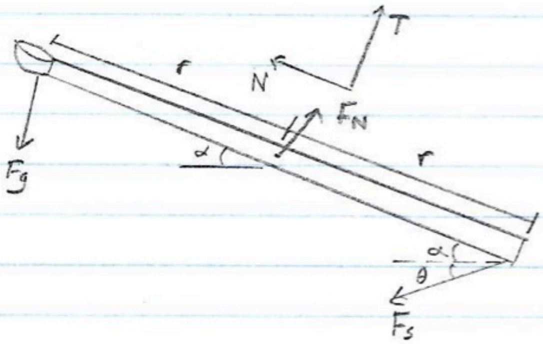
$$\boxed{I = 0.016971\,kg\cdot m^2}$$

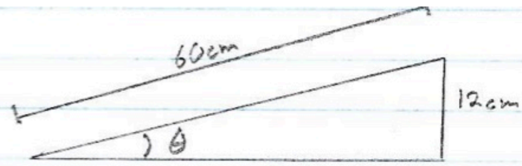
→ Plugging into ω^2 equtión gives...

$$\boxed{\omega^2 = \frac{2M(S)}{0.016971\,kg\cdot m^2}}$$

With this equation for angular velocity, the moments at the two different points needed to be calculated.

Start of Launch





$\theta = \sin^{-1} \frac{12}{60} = \underline{11.54^\circ}$

$\alpha = 16^\circ$ $m_{Lever} = 0.360\,Kg$

$\theta = 11.54^\circ$

$\theta_T = 16^\circ + 11.54^\circ = \underline{27.54^\circ}$

$\Sigma M = F_s \sin\theta \cdot r - F_N(0) - mg \cos\alpha \cdot r$

$\Sigma M = F_s \sin\theta \cdot 0.25m - 0.360Kg \cdot 9.81m/s^2 \cdot \cos 16^\circ \cdot 0.25m$

$= K \cdot \Delta x \sin 27.54^\circ \cdot 0.25m - 0.848697$

→ K value for one elastic tourniquet was determined to be 460N. Since there were 10 tourniquets, it will be assumed that the ten act as one so the K value is 460N×10 = $\underline{4600N/m}$

→ the Δx value is 6cm since when the elastic is streched it is 60cm, and when the lever arm is at 45° (launch position) the elastic is 54cm long.

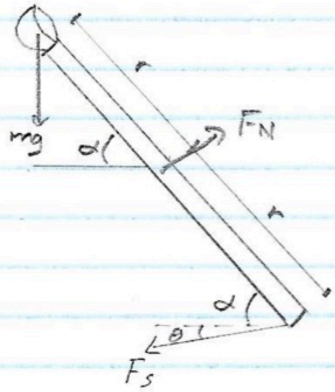
$\therefore M = 4600\frac{N}{m} \cdot 0.06m \cdot \sin 27.54^\circ \cdot 0.25m - 0.848697\,N\cdot m$

$M = \underline{31.05467\,N\cdot m}$

$= \boxed{31.05\,N\cdot m}$

The moment values and "S" values for the initial point and the final point of the launching sequence were calculated and substituted into the equation for angular velocity.

Launch Point (when lever arm makes 45° angle with horizontal)



$\alpha = 45°$
$\theta = 2.46°$
$\theta_T = \alpha + \theta = 45° + 2.46° = \underline{47.46°}$

$\sum M = F_s \sin\theta_T \cdot r - F_N(0) - mg\cos\alpha \cdot r$
$= 4600\frac{N}{m} \cdot 0.06m \cdot \sin 47.46° - (0.360Kg)(9.81m/s^2)(\cos 45°)$
$= (203.3583\,N \cdot 0.25m) - (2.497218\,N \cdot 0.25m)$
$\boxed{M = 50.2152705\ N\cdot m}$

Velocities

At Start of Launch

$\omega^2 = \frac{2M(0.0424m)}{0.016971\ kg\cdot m^2}$
$= \frac{2(31.05N\cdot m)(0.0424m)}{0.016971\ Kg\cdot m^2} = \frac{2.63343\ N\cdot m^2}{0.016971\ Kg\cdot m^2}$

$\sqrt{\omega^2} = \sqrt{155.1727\ rad/s^2}$
$\omega = 12.456\ rad/s$

$\therefore V = \omega \cdot r$
$= 12.456\ rad/s \cdot 0.25m = \boxed{3.1139\ m/s}$

At Launch Point

$$\omega^2 = \frac{2M\,(0.1266m)}{0.016971\ Kg\cdot m^2} = \frac{2\cdot(50.215\ N\cdot m)(0.1266m)}{0.016971\ Kg\cdot m^2}$$

$$\omega^2 = \frac{12.65418\ N\cdot m^2}{0.016971\ Kg\cdot m^2} = 749.19018\ rad/s^2$$

$$\omega = \sqrt{749.19018\ rad/s^2} = 27.371\ rad/s$$

$$\therefore\ V = \omega\cdot r$$

$$= 27.371\ rad/s \cdot 0.25m = \boxed{6.84\ m/s}$$

Solving for the angular velocity at the two points, the equation $V_{\emptyset} = \omega$r was solved to find the velocity of the snowball at the initial point and at the final point of launch of the launch sequence.

The velocity at the point of launch in the Excel file is $10.98m/s$. Compared to the $6.84\ m/s$ calculated above, a reason for the value calculated above being off by close to $4m/s$ could be due to the fact that there is friction in the bolt where the lever arm rotated about that was not accounted for. This however is unlikely to be the reason because the bolt use seemed to have little to no friction with the lever arm. Even is the friction was accounted for, it is not expected to have varied the answer by $4m/s$. The mostly likely error that caused this answer to not be exact is the path length calculated.

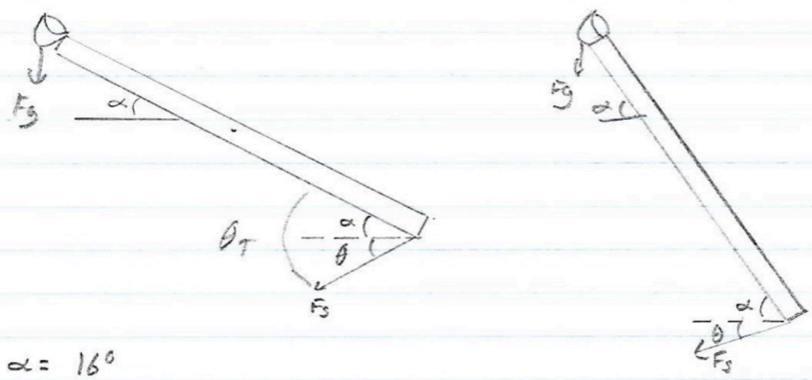
The velocity calculated at the initial point however was off by only $2m/s$. This could be due to the path variable “S” that was used in this calculation, causing the value of velocity to be incorrect.

2) Acceleration

Acceleration

→ acceleration from A to B.

A (Start of Launch)   B (Launch Point)



$\alpha = 16°$
$\theta = 11.54°$
$\theta_T = 27.54°$

$\alpha = 45°$
$\theta = 2.46°$
$\theta_T = 47.46°$

→ Using the angular velocities calculated previously, and using the times from the excel file that correspond to the two different points, the equation shown below can be used to find angular acceleration.

$$\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

$\omega_2 = 27.371 \text{ rad/s}$
$\omega_1 = 12.436 \text{ rads}$

$t_2 = 0.099s$
$t_1 = 0.033s$

$$\therefore \alpha = \frac{27.371 \text{ rad/s} - 12.436 \text{ rad/s}}{0.099s - 0.033s}$$

$$= \frac{14.915 \text{ rad/s}}{0.066s \text{ s}}$$

$$= \boxed{225.9848 \text{ rad/s}^2}$$

→ Now, using the angular acceleration use calculated, the tangent acceleration will be calculated using:

$$a_T = \alpha \cdot r$$

$\therefore a_T = 225.984\ rad/s^2 \cdot 0.25m$

$a_T = 56.496 m/s^2$

∴ The acceleration from when the launcher begins to accelerate to ball, until the snowball leaves the snowball holder is $56.496\ m/s^2$.

The average acceleration from the Excel file from when the ball is at the initial point to the final point of launch is $73.26 m/s^2$. The reason why these values are not equal could be due to the inaccuracy of the data in the excel file. Another reason why these values were not equal is likely due to the assumptions made when calculating the acceleration above. The assumptions were constant acceleration along path of travel and a constant force applied by the elastic.

Improvements

The possible improvements that could be made to this device to make it perform better would be improvements to increase the distance that the design could throw a snowball. The improvements that could be made include increasing the force caused by the elastic tourniquets on the lever arm and increasing the length of the lever arm. Focusing on increasing the length of the lever arm, this would increase the length of the path that the snowball takes when it is in the device. In turn, increasing the amount of time the force caused by the elastic tourniquets. This would ultimately increase the velocity that is given to the snowball while it is still being acted on by the force of the device. Below is a calculation proving this.

New Path

$\Delta\theta = 45° - 16° = 29°$ (assuming launch is at 45°)

$\rightarrow \frac{29°}{360°} = 0.0806$

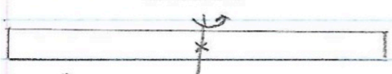
$r = 50cm = \underline{0.5m}$ $\rightarrow$ Circumference $= 2\pi r = 2\pi \cdot 0.5m$
$= \underline{3.14m}$

$\therefore$ Path $= 0.0806 \cdot 3.14m = \boxed{0.2532m}$

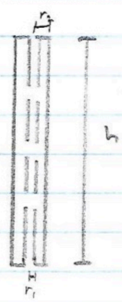
Velocity at Launching Point (if lever arm length doubled)

$I = I_{arm} + I_{holder} + I_{snowball}$

$I_{arm} + I_{holder}$



$I_y = \frac{1}{12}(m(3(r_2^2 + r_1^2) + h^2))$



$\therefore I_y = \frac{1}{12}(0.72\,kg(3(0.017^2 m + 0.013^2 m) + 1^2 m)$

$= \underline{0.0600\ kg \cdot m^2}$

$I_{snowball} = 0.009471\ kg \cdot m^2$ (same as before)

$\therefore I = 0.009471 + 0.0600 = \underline{0.06947\ kg \cdot m^2}$

Plugging into: $\omega^2 = \frac{2M(0.2532m)}{0.06947\ kg \cdot m^2}$

$\omega^2 = \frac{2(50.2152\ N \cdot m)(0.2532m)}{0.06947\ kg \cdot m^2} = \frac{25.428}{0.06947\ kg}$

$\omega^2 = 366.042\ rad/s^2$

$\therefore \underline{\omega} = 19.13\ rad/s$

Using:

$$V = \omega \cdot r$$

$$V = 19.1322\ rad/s \cdot 0.5m = \boxed{9.5661\ m/s}$$

As shown from the calculation above, doubling the length of the lever arm increasing the velocity of the snowball at the point of launch by $2.73m/s$.

Recommendations

A recommendation I would make if I were to recreate this design would be to figure out a way have the elastic tourniquets go straight to the grip when the device is ready to fire (sketch in appendix C). This could be done but shorting the grip. Doing this would make the calculation of velocity and acceleration easier and more accurate. It would make it more accurate due to the fact that the angle between the horizontal and the elastic at the grip would not change. The change in this angle was not accounted for when calculating the velocity and could have caused my velocity and acceleration to be wrong. I would also increase the length of the lever arm. By doing this it would make the launch velocity greater, as proved in the "Improvements" section, and therefore launch the snowball a farther distance.

What Was Learned

The point of this design was to dominate in close quarters combat, and it proved it would due to its fast fire rate, okay accuracy and mediocre range. However, different recommendations for improvements could be made to this device in order to get different results. An example of this would be in order to get the device to throw a snowball a far distance, implement a long lever arm into the design. If a more accurate version of this design is desired, create a stand for the device so that the design is balanced. If both of these elements are desired, incorporate a long lever arm and a stand into the design. Depending on what type of device is desired, the design of the device will vary. It is very difficult to make a design that can throw far, accurate and has a quick fire rate.

With regards to course content, I learned how to use dynamics when analyzing a design. In particular, I learned different ways to calculate accelerations and velocities from moments of inertia and angular velocities. I began to understand how equations are influenced by changing the variables within them and how to apply these calculations in the design phase to make the design as effective as possible.

Appendix

Appendix A – Device Sketches

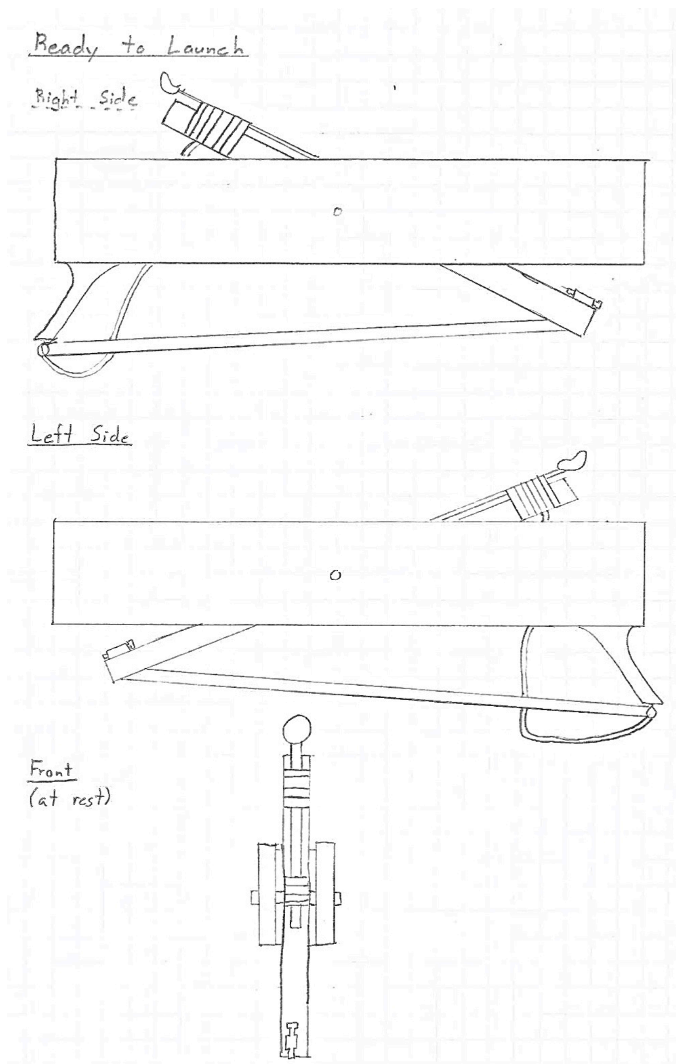


Figure 1.1: Sketch of the Launcher at ready positions.

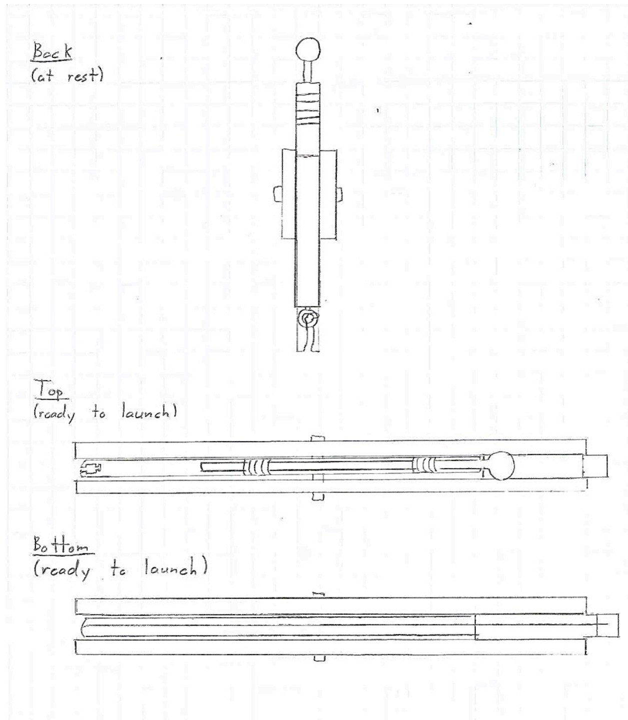


Figure 1.2: *Sketch of the Launcher at rest and ready positions.*

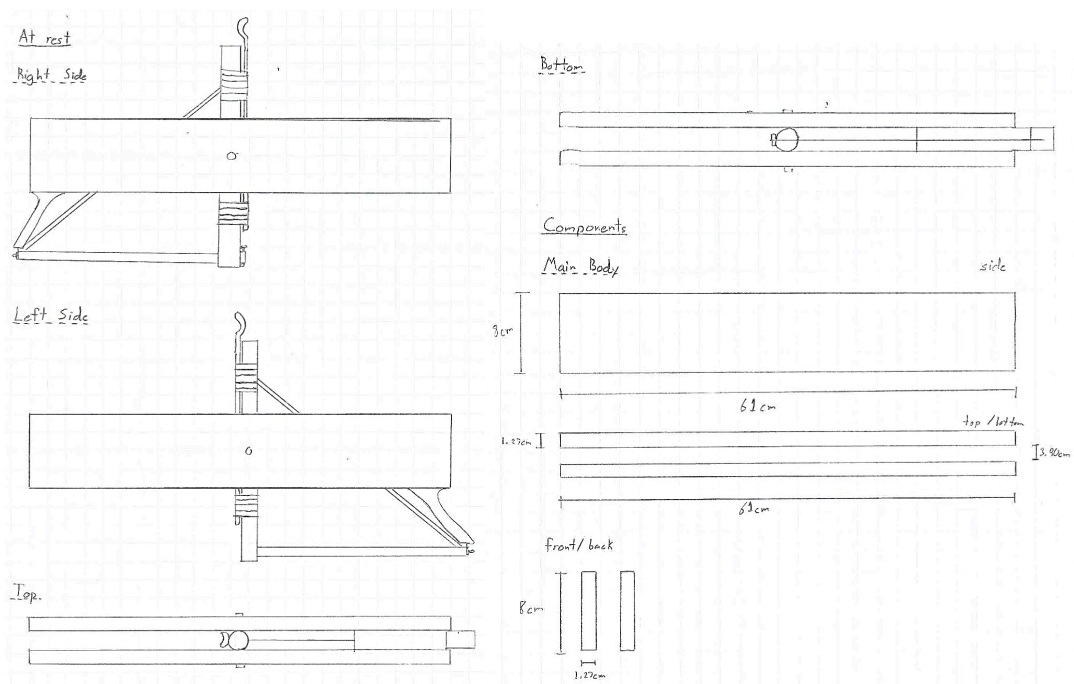
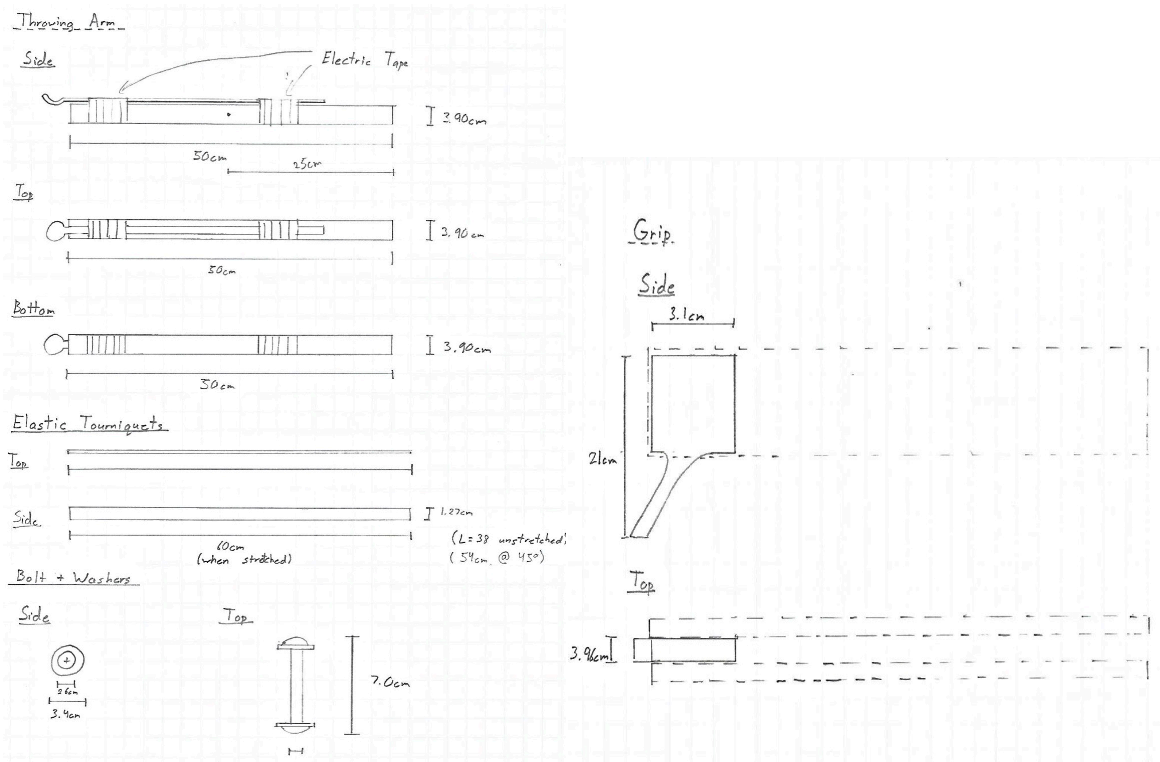
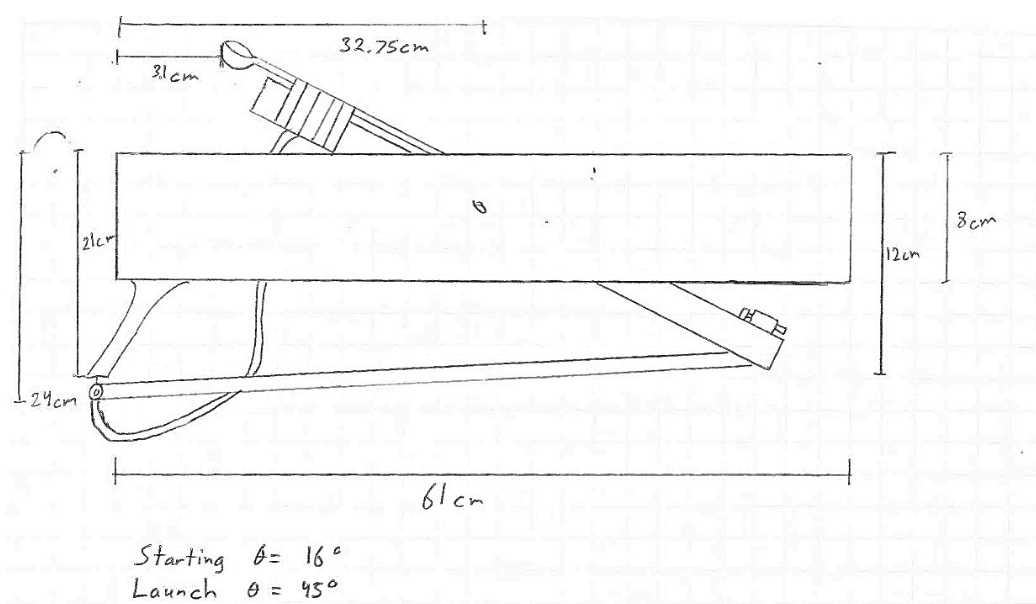


Figure 1.2 and 1.3: *Sketch of the launcher at rest position and sketches of individual components*

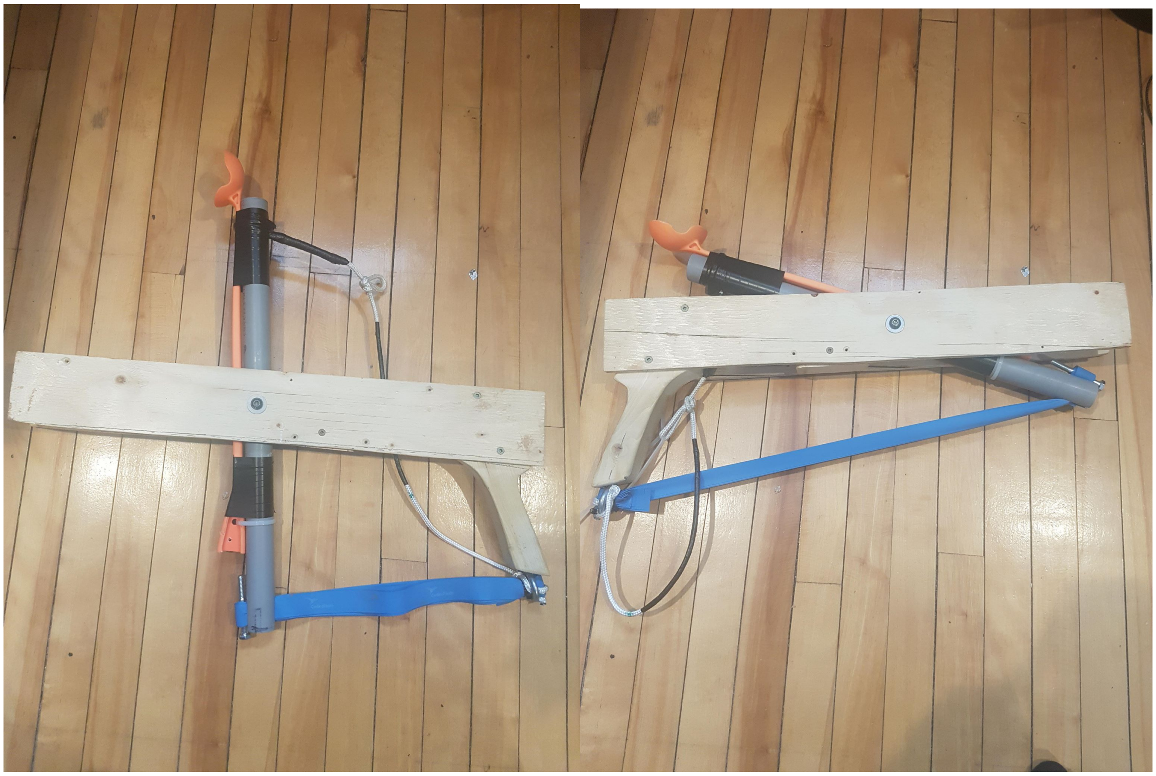


<u>Figure 1.4 and 1.5:</u> Additional sketches of the launcher components.



<u>Figure 1.6:</u> Additional sketch of the launcher with dimensions.

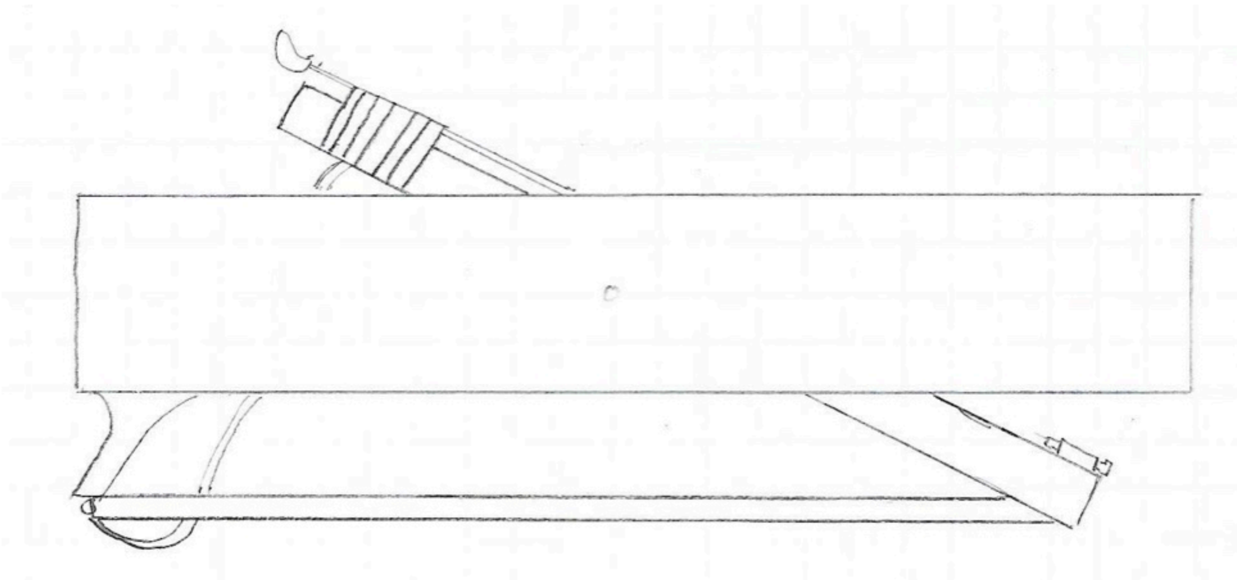
Appendix B – Device Photos



<u>Figure 2.1 and 2.2:</u> Photos taken by B. Jamieson of final design taken when at rest, and when the launcher is ready to be launched.

<u>Figure 2.3:</u> An additional Photo of the launcher from the back

Appendix C – Recommended Design Sketch



<u>Figure 3.1:</u> A sketch of the recommended design.