APPENDIX H

FILTER DESIGN MATERIAL

This appendix provides filter design tables and circuits that supplement the material presented in Chapter 14.

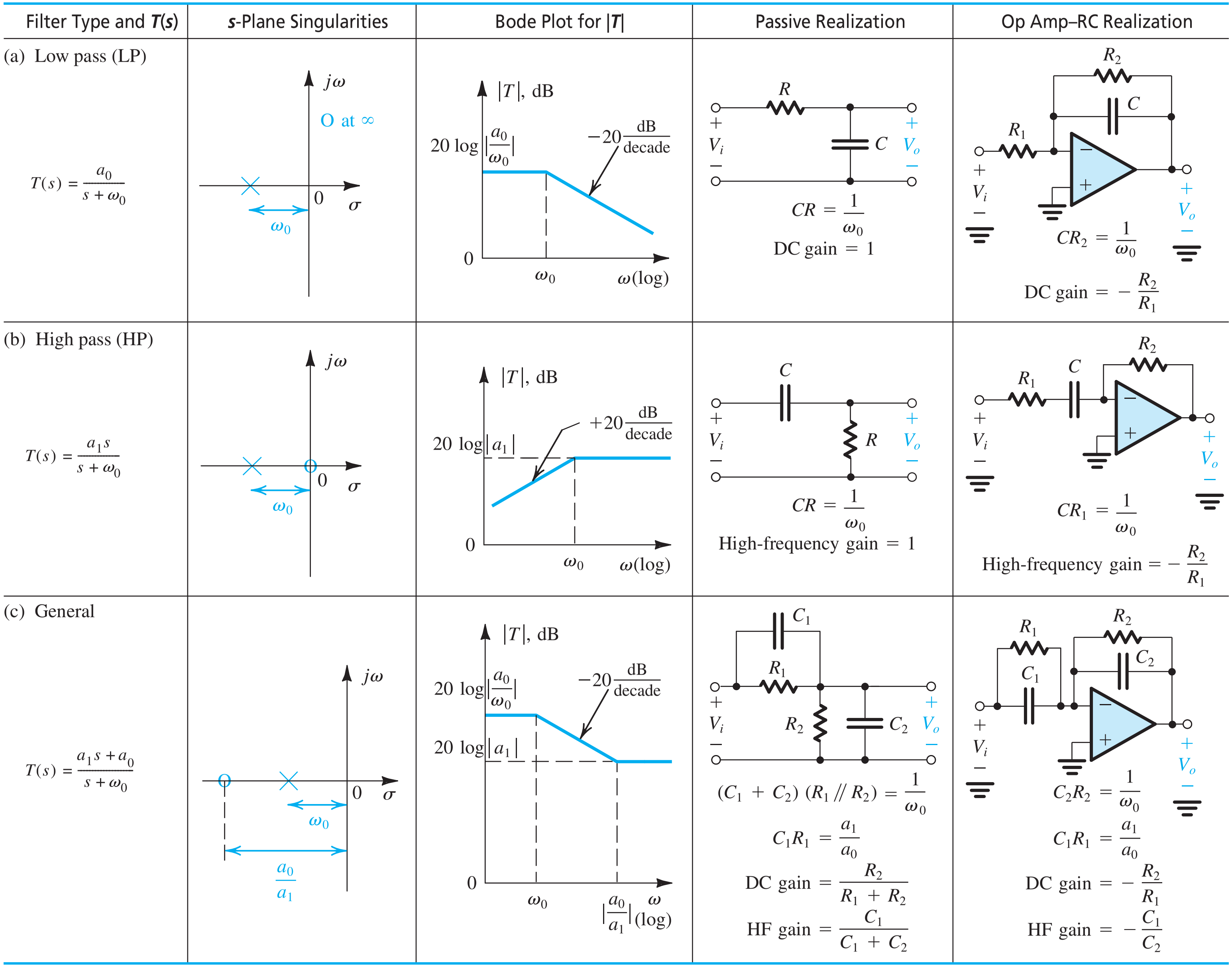


Figure H.1 First-order filters.

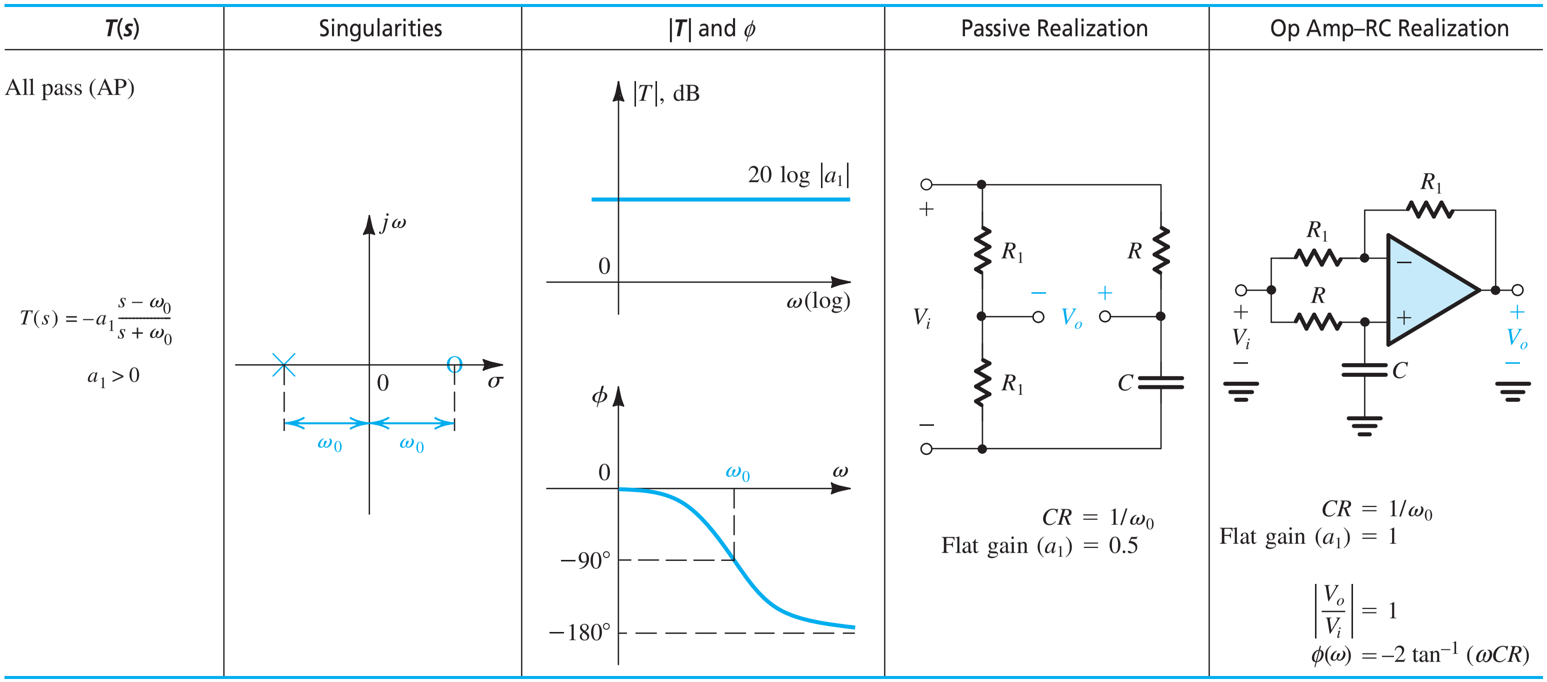


Figure H.2 First-order all-pass filter.

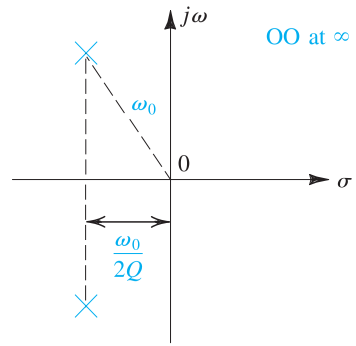
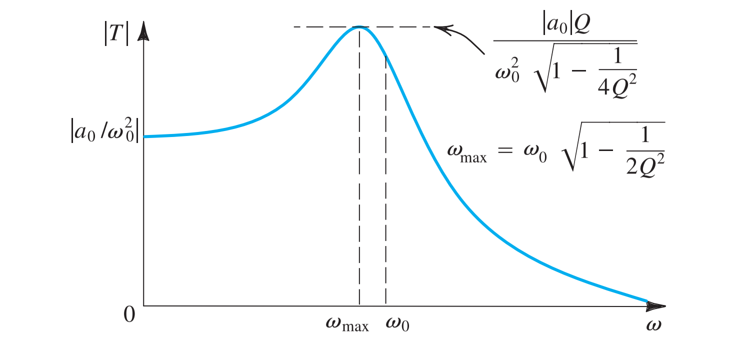
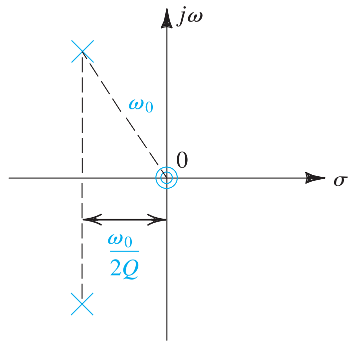
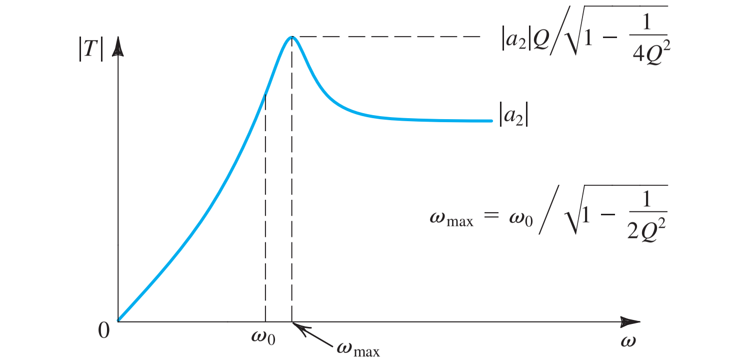
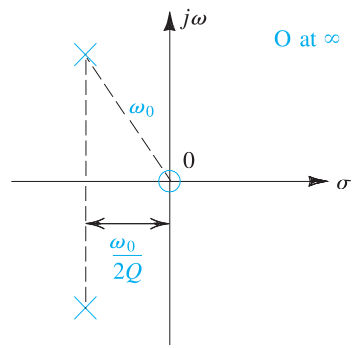
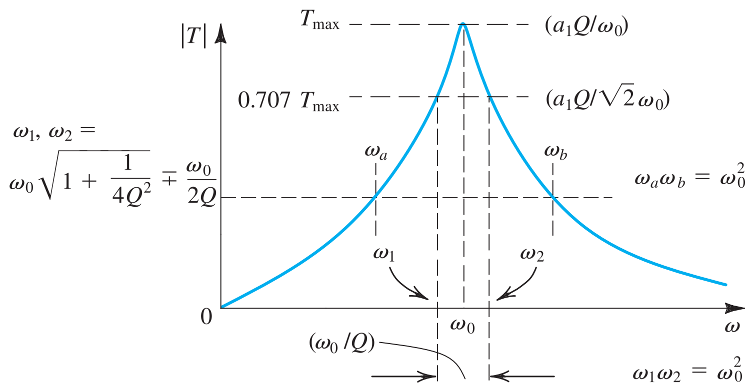
Filter Type and *T(s)*	*s*-Plane Singularities	|*T*|
(a) Low pass (LP) $T(s) = \dfrac{a_0}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ DC gain $= \dfrac{a_0}{\omega_0^2}$		
(b) High pass (HP) $T(s) = \dfrac{a_2 s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ High-frequency gain $= a_2$		
(c) Bandpass (BP) $T(s) = \dfrac{a_1 s}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ Center-frequency gain $= \dfrac{a_1 Q}{\omega_0}$		

Figure H.3 Second-order filter functions.

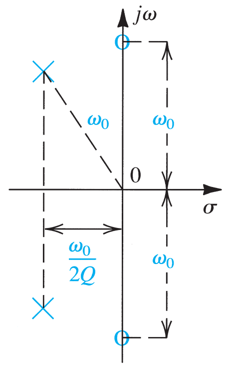
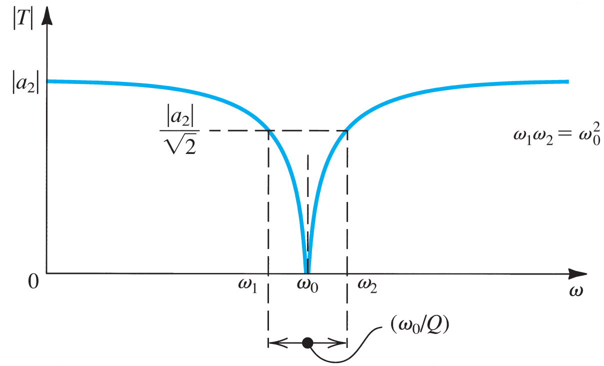
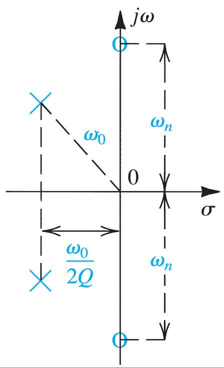
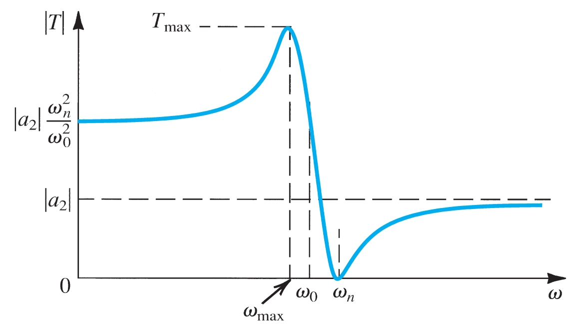
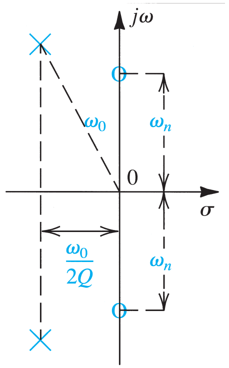
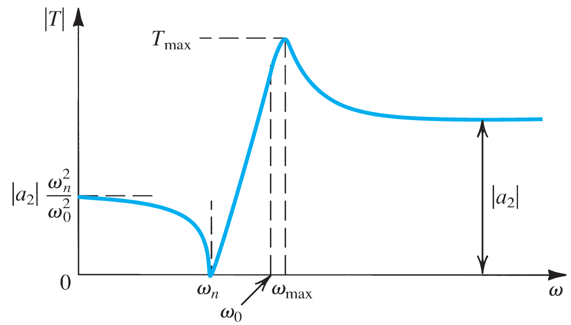
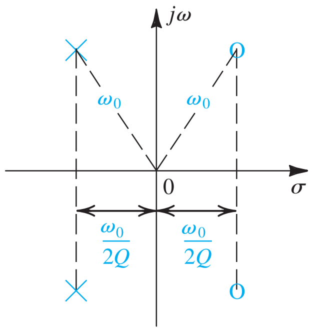
Filter Type and $T(s)$	s-Plane Singularities	$\lvert T\rvert$
(d) Notch $T(s) = a_2 \dfrac{s^2 + \omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ DC gain = High-frequency gain $= a_2$		
(e) Low-pass notch (LPN) $T(s) = a_2 \dfrac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ $\omega_n \geq \omega_0$ DC gain $= a_2 \dfrac{\omega_n^2}{\omega_0^2}$ High-frequency gain $= a_2$		 $\omega_{max} = \omega_0 \sqrt{\dfrac{\left(\dfrac{\omega_n^2}{\omega_0^2}\right)\left(1 - \dfrac{1}{2Q^2}\right) - 1}{\dfrac{\omega_n^2}{\omega_0^2} + \dfrac{1}{2Q^2} - 1}}$
(f) High-pass notch (HPN) $T(s) = a_2 \dfrac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ $\omega_n \leq \omega_0$ DC gain $= a_2 \dfrac{\omega_n^2}{\omega_0^2}$ High-frequency gain $= a_2$		 $T_{max} = \dfrac{\lvert a_2\rvert \, \lvert\omega_n^2 - \omega_{max}^2\rvert}{\sqrt{(\omega_0^2 - \omega_{max}^2)^2 + \left(\dfrac{\omega_0}{Q}\right)^2 \omega_{max}^2}}$

Figure H.3 *continued*

(g) All pass (AP)

$$T(s) = a_2 \frac{s^2 - s\frac{\omega_0}{Q} + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Flat gain $= a_2$



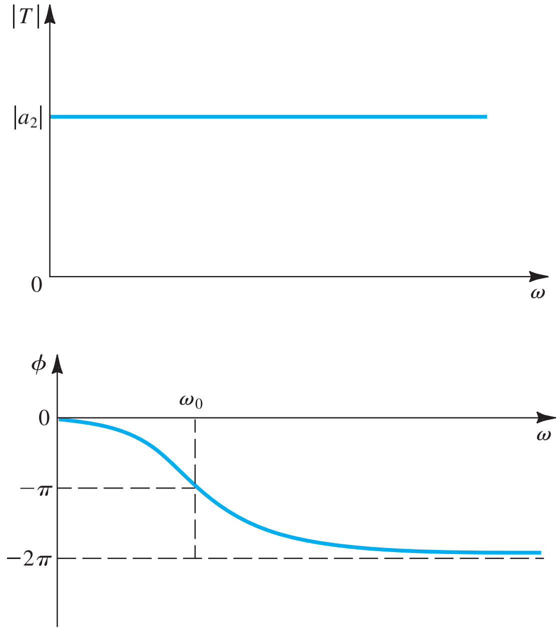


Figure H.3 *continued*

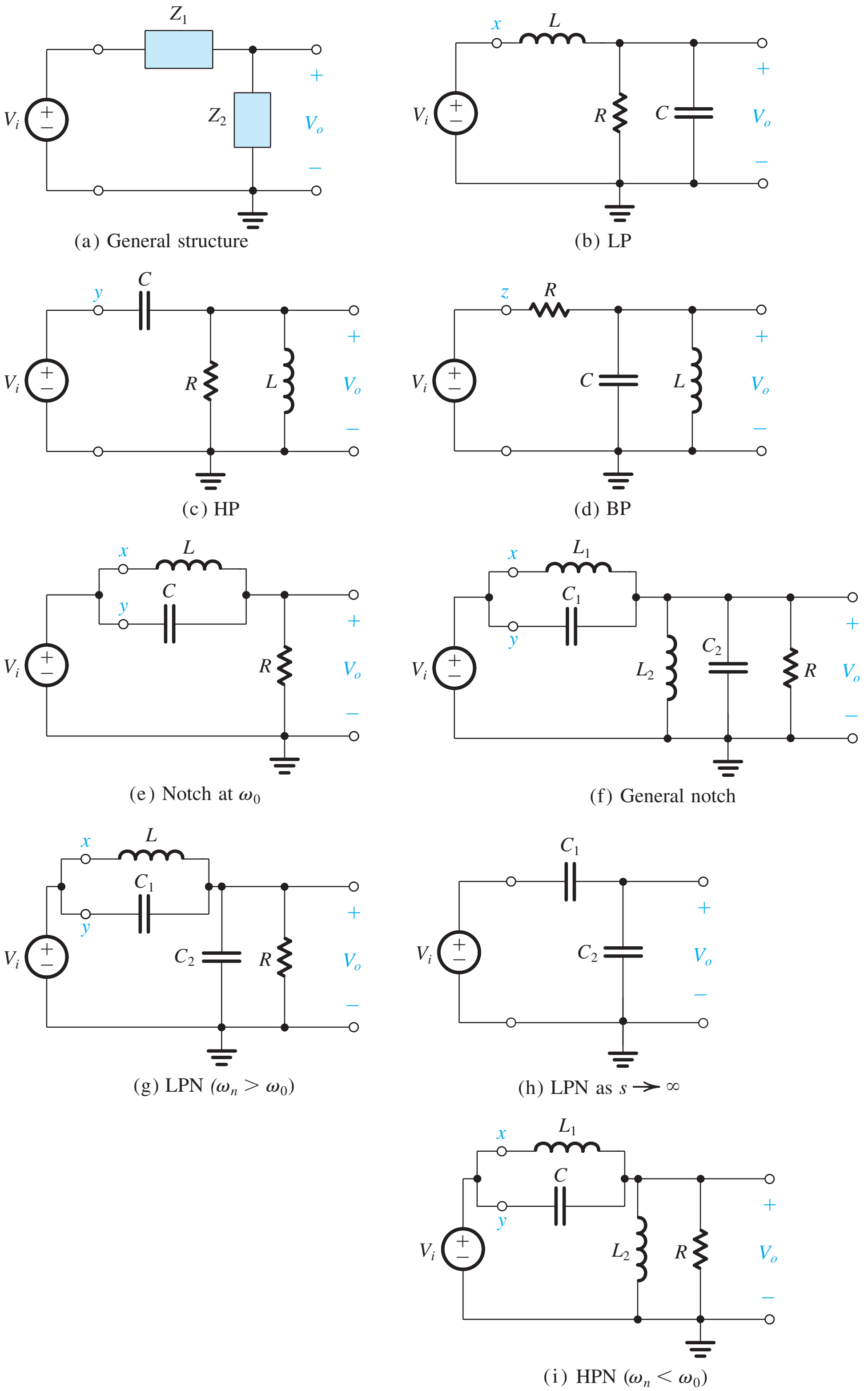


Figure H.4 Realization of various second-order filter functions using the LCR resonator of Fig. 14.18(a): **(a)** general structure, **(b)** LP, **(c)** HP, **(d)** BP, **(e)** notch at ω_0, **(f)** general notch, **(g)** LPN $(\omega_n \geq \omega_0)$, **(h)** LPN as $s \rightarrow \infty$, **(i)** HPN $(\omega_n < \omega_0)$.

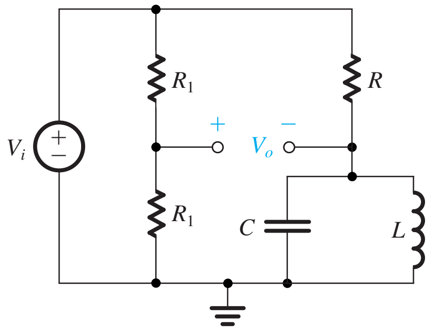


Figure H.5 Realization of the second-order all-pass transfer function using a voltage divider and an LCR resonator.

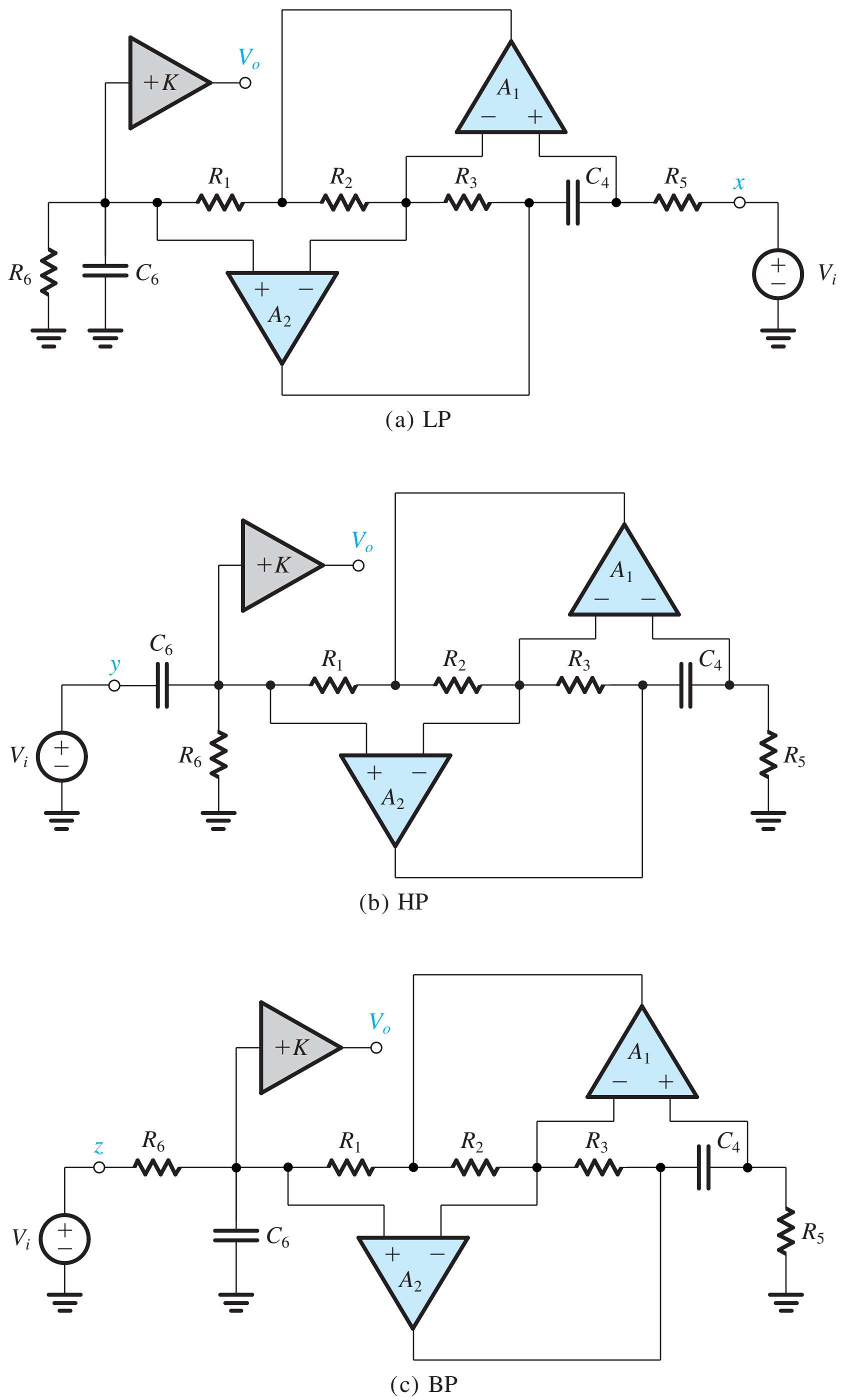
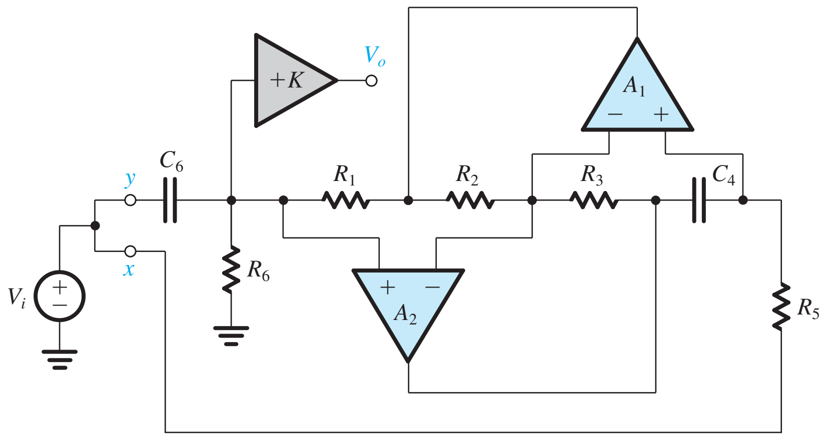
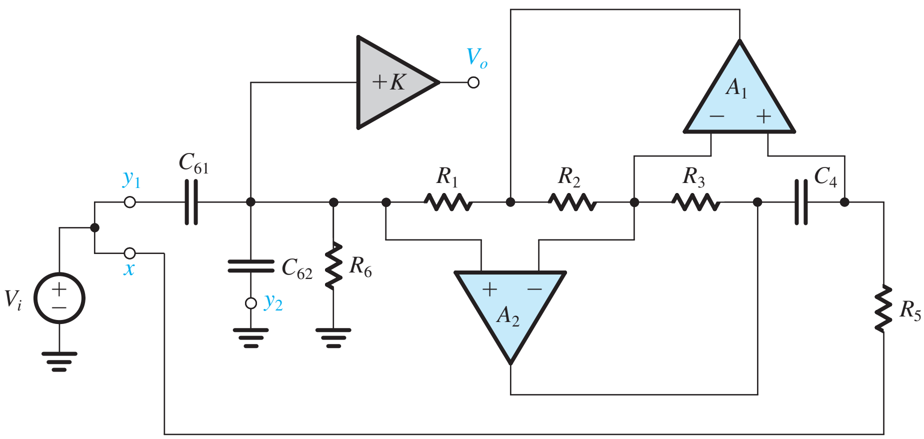


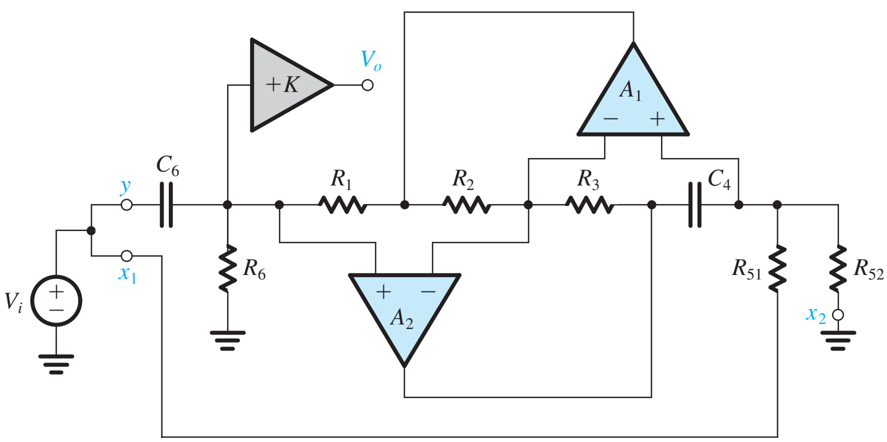
Figure H.6 Realizations for the various second-order filter functions using the op amp–RC resonator of Fig. 14.21(b): **(a)** LP, **(b)** HP, **(c)** BP. The circuits are based on the LCR circuit in Fig. 14.18. Design considerations are given in Table H.1.



(d) Notch at ω_0

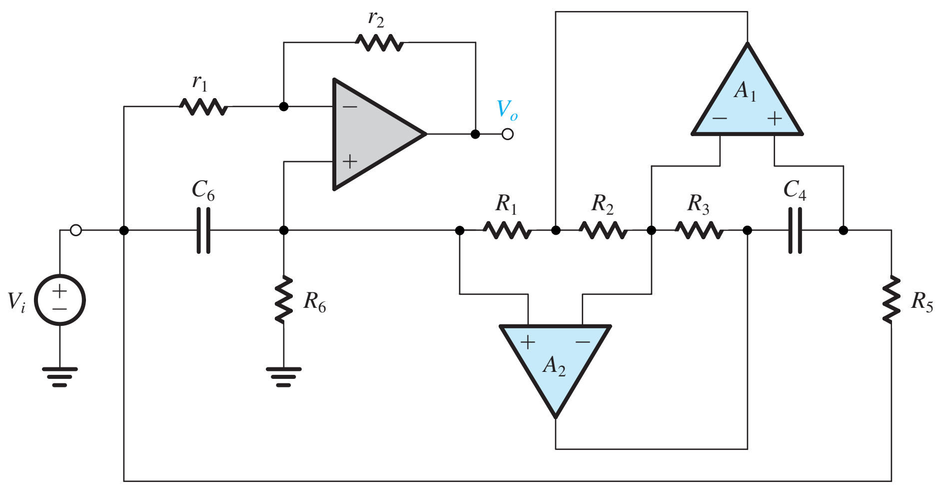


(e) LPN, $\omega_n \geq \omega_0$



(f) HPN, $\omega_n \leq \omega_0$

Figure H.6 *continued* **(d) Notch at** ω_0; **(e) LPN,** $\omega_n \geq \omega_0$; **(f) HPN,** $\omega_n \leq \omega_0$.



(g) All-pass

Figure H.6 *continued* **(g) All pass.**

Table H.1 Design Data for the Circuits of Fig. H.6

Circuit	Transfer Function and Other Parameters	Design Equations
Resonator Fig. 14.18(a)	$\omega_0 = 1/\sqrt{C_4 C_6 R_1 R_3 R_5 / R_2}$ $Q = R_6 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_1 R_3 R_5}}$	$C_4 = C_6 = C$ (practical value) $R_1 = R_2 = R_3 = R_5 = 1/\omega_0 C$ $R_6 = Q/\omega_0 C$
Low-pass (LP) Fig. H.6(a)	$T(s) = \dfrac{KR_2/C_4 C_6 R_1 R_3 R_5}{s^2 + s\frac{1}{C_6 R_6} + \frac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	$K =$ DC gain
High-pass (HP) Fig. H.6(b)	$T(s) = \dfrac{Ks^2}{s^2 + s\frac{1}{C_6 R_6} + \frac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	$K =$ High-frequency gain
Bandpass (BP) Fig. H.6(c)	$T(s) = \dfrac{Ks/C_6 R_6}{s^2 + s\frac{1}{C_6 R_6} + \frac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	$K =$ Center-frequency gain
Regular notch (N) Fig. H.6(d)	$T(s) = \dfrac{K\left[s^2 + (R_2/C_4 C_6 R_1 R_3 R_5)\right]}{s^2 + s\frac{1}{C_6 R_6} + \frac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	$K =$ Low- and high-frequency gain

Low-pass notch (LPN) Fig. H.6(e)	$T(s) = K\dfrac{C_{61}}{C_{61}+C_{62}} \times \dfrac{s^2 + (R_2/C_4C_{61}R_1R_3R_5)}{s^2 + s\dfrac{1}{(C_{61}+C_{62})R_6} + \dfrac{R_2}{C_4(C_{61}+C_{62})R_1R_3R_5}}$ $\omega_n = 1/\sqrt{C_4C_{61}R_1R_3R_5/R_2}$ $\omega_0 = 1/\sqrt{C_4(C_{61}+C_{62})R_1R_3R_5/R_2}$ $Q = R_6\sqrt{\dfrac{C_{61}+C_{62}}{C_4}\dfrac{R_2}{R_1R_3R_5}}$	$K =$ DC gain $C_{61}+C_{62} = C_6 = C$ $C_{61} = C(\omega_0/\omega_n)^2$ $C_{62} = C - C_{61}$
High-pass notch (HPN) Fig. H.6(f)	$T(s) = K\dfrac{s^2 + (R_2/C_4C_6R_1R_3R_{51})}{s^2 + s\dfrac{1}{C_6R_6} + \dfrac{R_2}{C_4C_6R_1R_3}\left(\dfrac{1}{R_{51}}+\dfrac{1}{R_{52}}\right)}$ $\omega_n = 1/\sqrt{C_4C_6R_1R_3R_{51}/R_2}$ $\omega_0 = \sqrt{\dfrac{R_2}{C_4C_6R_1R_3}\left(\dfrac{1}{R_{51}}+\dfrac{1}{R_{52}}\right)}$ $Q = R_6\sqrt{\dfrac{C_6}{C_4}\dfrac{R_2}{R_1R_3}\left(\dfrac{1}{R_{51}}+\dfrac{1}{R_{52}}\right)}$	$K =$ High-frequency gain $\dfrac{1}{R_{51}}+\dfrac{1}{R_{52}} = \dfrac{1}{R_5} = \omega_0 C$ $R_{51} = R_5(\omega_0/\omega_n)^2$ $R_{52} = R_5\Big/\left[1-(\omega_n/\omega_0)^2\right]$
All-pass (AP) Fig. H.6(g)	$T(s) = \dfrac{s^2 - s\dfrac{1}{C_6R_6}\dfrac{r_2}{r_1} + \dfrac{R_2}{C_4C_6R_1R_3R_5}}{s^2 + s\dfrac{1}{C_6R_6} + \dfrac{R_2}{C_4C_6R_1R_3R_5}}$ $\omega_z = \omega_0 \quad Q_z = Q(r_1/r_2) \quad$ Flat gain $= 1$	$r_1 = r_2 = r$ (arbitrary) Adjust r_2 to make $Q_z = Q$

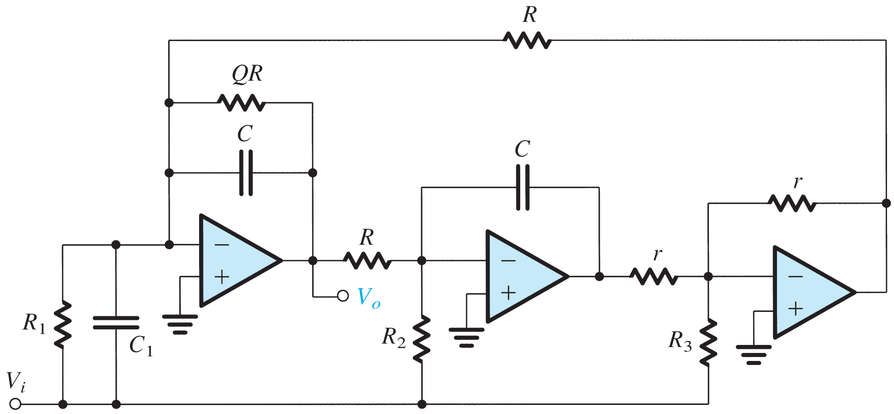


Figure H.7 The Tow–Thomas biquad with feedforward. The transfer function of Eq. (14.70) is realized by feeding the input signal through appropriate components to the inputs of the three op amps. This circuit can realize all special second-order functions. The design equations are given in Table H.2.

Table H.2 Design Data for the Circuit in Fig. H.7(and Fig. 14.26)

All cases	$C =$ arbitrary, $R = 1/\omega_0 C$, $r =$ arbitrary
LP	$C_1 = 0$, $R_1 = \infty$, $R_2 = R$/dc gain, $R_3 = \infty$
Positive BP	$C_1 = 0$, $R_1 = \infty$, $R_2 = \infty$, $R_3 = Qr$/center-frequency gain
Negative BP	$C_1 = 0$, $R_1 = QR$/center-frequency gain, $R_2 = \infty$, $R_3 = \infty$
HP	$C_1 = C \times$ high-frequency gain, $R_1 = \infty$, $R_2 = \infty$, $R_3 = \infty$
Notch (all types)	$C_1 = C \times$ high-frequency gain, $R_1 = \infty$, $R_2 = R(\omega_0/\omega_n)^2$/high-frequency gain, $R_3 = \infty$
AP	$C_1 = C \times$ flat gain, $R_1 = \infty$, $R_2 = R$/gain, $R_3 = Qr$/gain