Chapter 4

Sensitivity Analysis

Uncertainty in the data is one of the largest differences between typical end-of-chapter problems and the more realistic problems of cases and the real world. While many management problems rely on quantitative data, rarely are those data exact. Estimation is required for some current values, and even more uncertainty is introduced by forecasting likely or possible future values. Engineering economy's emphasis on future cash flows and its application to projects at the preliminary design stage requires that this problem be specifically addressed.

Deterministic approximations ignore this uncertainty, which can be addressed through risk analysis, simulation, and sensitivity analysis. This last approach is often easiest, and it may also do the most to develop a "feel" for the problem. The simplest form of sensitivity analysis is to ask, "What if?" Different values for different scenarios can be entered into the spreadsheet to see how and if the recommended decision changes.

The next level of sensitivity analysis is examining the impact of reasonable changes in "base case" assumptions. As input to the sensitivity analysis, for each variable we need to know its most likely value, and lower and upper limits for reasonable changes in that value. From this we can calculate:

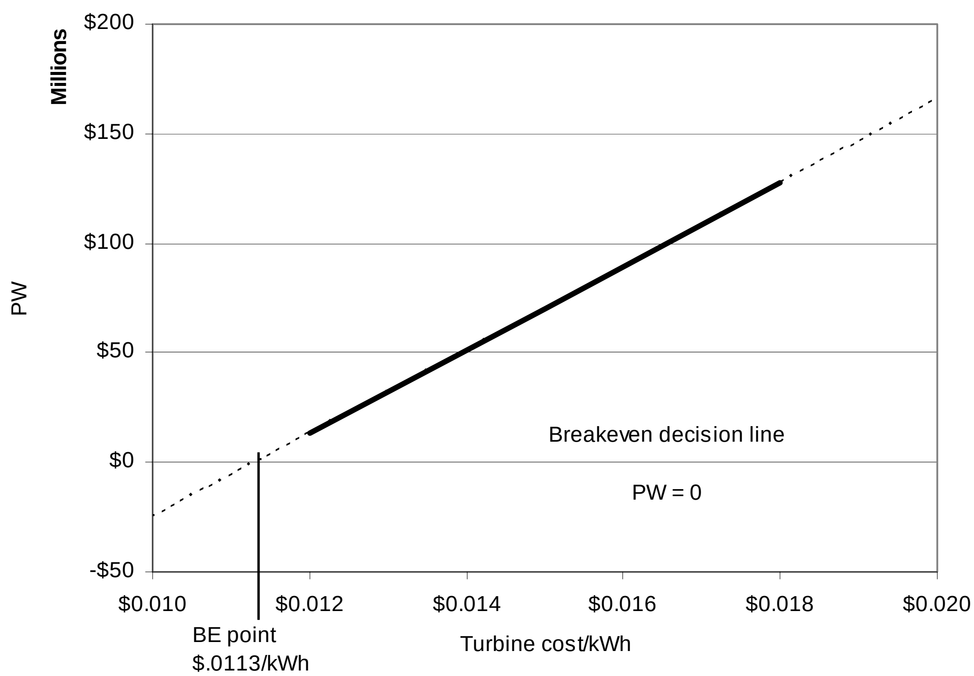
- The unit impact of these changes on the present worth (PW) or other measure of quality
- The maximum impact of each variable on the present worth
- The amount of change required to crossover a breakeven decision line

A sensitivity analysis can be done using any measure of quality, for example, injuries per year, net sales revenue, or internal rate of return. Present worth was used in Chapter 2 for the example case, and it is used here.

Breakeven Charts

A breakeven chart is constructed by holding all variables constant at their base case values and changing one variable to find its breakeven value, which is where the PW equals $0. This can be drawn without the lower and upper limits for reasonable change in the variable, but it is more useful if those are included. Figure 4-1 is an example. In this case, a heavy line weight is used for those values that are within the limits, and a light dashed line is used to extend the relationship to the breakeven value of $.0113/kWh. For graphing in a spreadsheet, this is done by using a second series of points for the heavy weight line.

Figure 4-1 Example Breakeven Chart for a Turbine's Cost per kWh

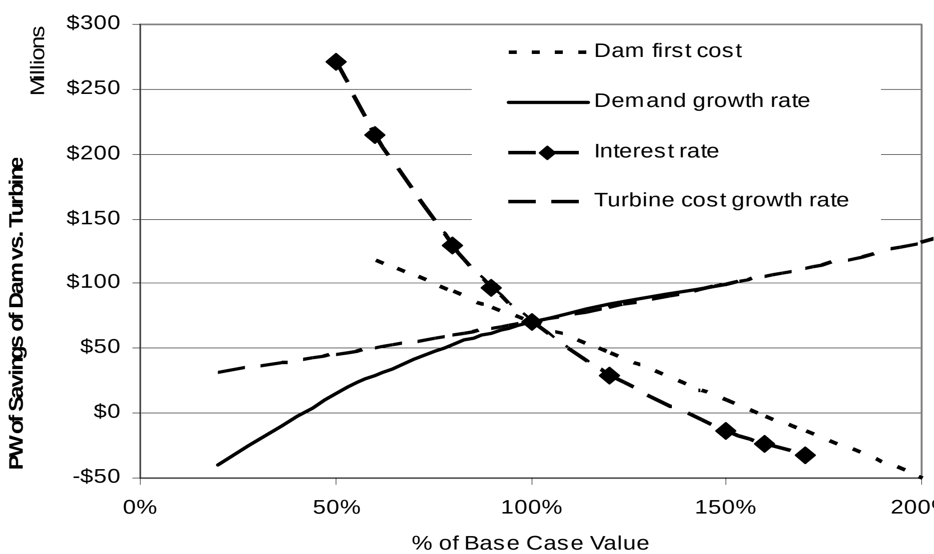


Relative Sensitivity Graph I – the Spiderplot

With a change in the *x*-axis, the information from a number of breakeven charts can be combined into a single relative sensitivity graph or spiderplot. For our example, this allows us to compare the relative sensitivity of the PW to changes in the growth rate in demand, the dam's first cost, the interest rate, etc. Because the "natural" units for these variables are all different (%, $, megawatt-hours/year), a common metric must be identified, for example, percentage of the base case value. (Note: This metric works poorly for variables with small or zero base case values. For these variables "natural" units work better.)

In the spiderplot, the variables are typically placed on the *x*-axis and the PW on the *y*-axis (see Figure 4-2). This orientation is chosen because the PW is a dependent variable.

Figure 4-2 Example Spiderplot



This *relative sensitivity graph* at its best allows the decision maker to quickly grasp:

- The range of reasonable values for each independent variable

- The unit impact of each independent variable on the PW (measured by the curve's slope)
- Breakeven values for variables whose curves cross the axis for PW = 0: 160% of the base case value for the dam's first cost, 140% of the base case value for the discount rate, and 40% of the base case value for the growth rate in demand

Combined these factors allow the decision maker to:

- Compare the maximum variation in PW for each variable
- Identify those changes that would support a different decision

At its worst, a relative sensitivity graph would mislead by distorting the scale to exaggerate or minimize slopes, by omitting any indication of the breakeven point, or by including too many or too few or the wrong variables.

The most common error is to graph all variables over the same range of percentage change. Imposing individual limits on the curves for each variable is crucial. Only then will this graph clearly describe the sensitivity of the PW to each variable. The heavy black line in Figure 4-1 corresponded to a ± 20% change in the cost per kWh for the turbine. If the curve for the turbine's cost were extended to ± 100% to match the *x*-axis limits of Figure 4-2, then the importance of this variable would be vastly overstated.

Just as a positive or negative PW cannot **determine** a decision by itself, but must instead be weighed against non-economic factors and the approximations of the model; so, too, must a breakeven curve or the crossing of PW = 0 be interpreted as a region of economic indifference.

Constructing a Spiderplot. If the *y*-axis value (PW in our example) can be expressed as a single formula, then this section describes how to build the table of values and draw the spiderplot. If the *y*-axis value can only be calculated using cash flow tables, then EXCEL DATA TABLES or PASTE SPECIAL/VALUES ONLY can be used to build the table of values and draw the spiderplot.

Even if the formula for the PW is very complex (cell A24 in Figure 2-6), the curve for each variable can be built by multiplying the variable's base case by a series of % values in the range between the minimum and maximum percentages of the base case. It is easier to build the table for all percentages of the base case and then to delete the "out-of-range" values. The cell addresses from Figure 2-6 are used as examples.

1. Create the data block of base case values and lower and upper limits. These limits are expressed as a % of the base case (E3:E13 and F3:F13).
2. Write the formula for the base case PW (cell A24) using absolute addresses (when the formula is copied, the addresses must not change).
3. Create the column headings (B27:I27) for the cash flow elements to be changed by copying from B3:B13 and using PASTE SPECIAL/TRANSPOSE. The row headings for the percentage points on the *x*-axis (A28:A39) are the different values listed in the lower and upper limits—listed in increasing order. Add a row for 100%—the base case.
4. Copy the formula for the base case into each cell of the top row of the table (B28:I28). Then edit the formula for *each* column, so that the cash flow element for that column is multiplied by the percentage value in column A. For example, *dam first cost* is the column head for B28:B39 and it is cell A3 in the data block. For row 28 in the *dam first cost* column every time A3 appears in the formula it is multiplied by cell A28. Repeat for the first formula in each column.
5. Copy the top row into the rest of the table. The values in the 100% row should all match the calculation of the base case PW. Check that the values are logical. For example does the PW go down as costs increase and go up as revenues increase?
6. Delete all entries that are beyond the limits for that column. For example, in this case the *dam's first cost* column should only have values between 60% and 200%. Similarly, the interest rate column should only have entries between 50% and 170%.
7. Create a graph by selecting the range A27:I39. The type should be XY. (If a line graph is selected, the *x*-axis values will be treated as labels and will be spaced evenly.) Each column of the table is a series. These series all have the same number of cells and *include the deleted entries.* For this example, the series for the dam's first cost is cells B28:B39, and the series for the *x*-axis is cells A28:A39.
8. If PW is the *y*-axis, add a variable that equals 0 for all *x*-axis values. If the *y*-axis is a B/C ratio, set the extra variable equal to 1. If the *y*-axis is the internal rate of return (IRR), set the extra variable equal to the minimum attractive rate of return. Add labels to the graph.

Typical Shapes to Expect. Understanding typical shapes for the curves for each variable is helpful in identifying when some kind of error has been made in building the economic model or in graphing the results.

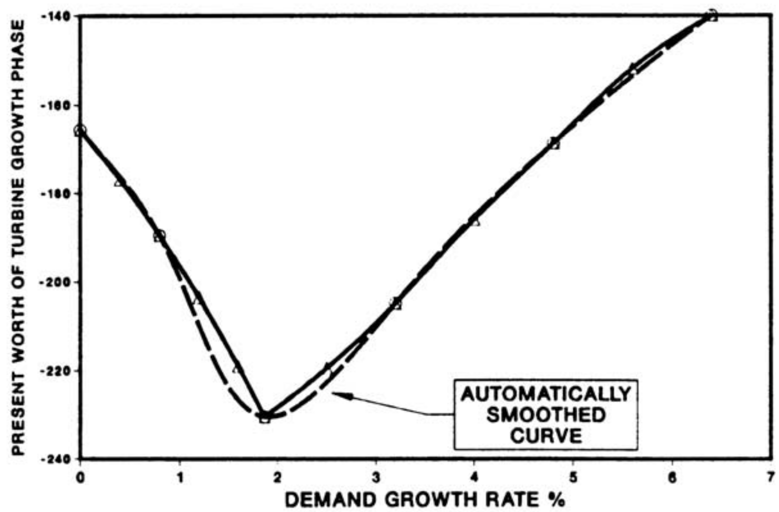
For example, first costs, periodic payments or receipts, and other parameters found outside of the compound interest factors are usually linearly related to the PW. Variables—such as the discount rate, inflation and other geometric gradients, the life of a machine, or the problem's horizon—are found in the time value of money factors, and they exhibit a curved relationship to PW.

Stated graphically these curves are usually convex or concave. Stated economically, for investments, the discount rate has negative and decreasing returns to scale, and the project life has positive and decreasing returns to scale. For loans, the positive and negative flip-flop, but they both still exhibit decreasing returns to scale.

The large majority of variables show either a straight line or a concave or convex curved relationship to PW. So an expectation of linearity or decreasing returns to scale may be useful, but caution is necessary. Consider the dam's capacity in the example case. For considering the amount of power generated annually by the dam, this variable is outside of the compound interest factors. However, the magnitude of the dam's capacity also determines the number of years until this capacity is exceeded, which is inside some of the interest factors—so the net result is a complex curved relationship.

The curve for the PW, as a function of the demand's growth rate, is even more complex. (The demand's growth rate impacts the length of time until the dam's capacity is exceeded *and* the equivalent discount rate.) In examining this relationship, the smooth curve in Figure 4-3 was produced. The smoothing was, however, an artifact of the computer program being used. In Excel the choice is between a smooth curve and straight line segments. To produce the true curve with a sharp point, separate data series must be plotted on each side of that point with curve smoothing for each.

Figure 4-3 Dangers of Automatic Curve Smoothing



Relative Sensitivity Graph II – the Tornado Diagram

Economic models will often have far more variables than can be effectively shown on a spiderplot. Dozens of variables can be shown on a tornado diagram, such as Figure 4-4 from Chapter 2.

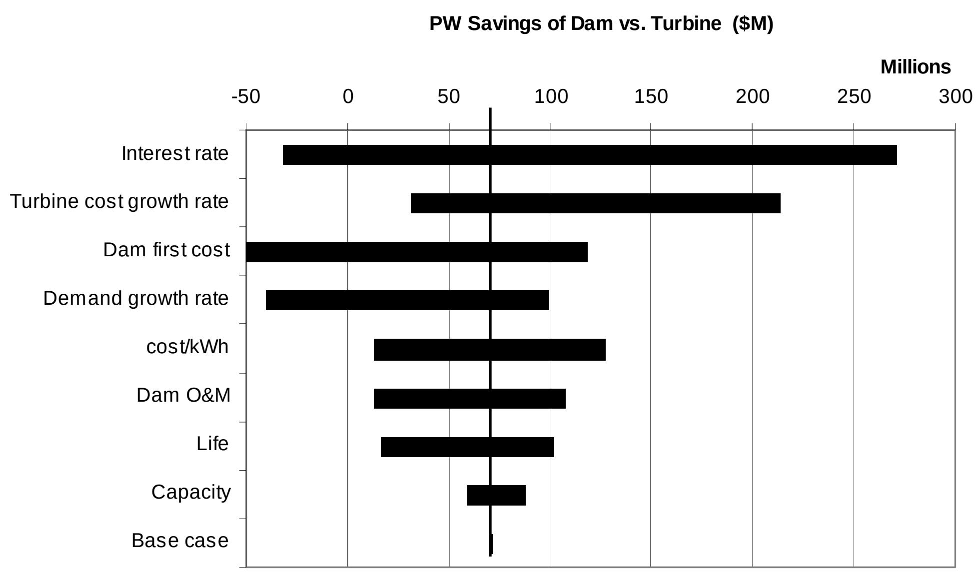
A tornado diagram does not show the actual relationship of the variables to the PW, only their possible impact on the PW. Thus, it doesn't show as much information, but it is easier to understand. It can also display the relative sensitivity of the PW to *all* variables. Figure 4-4 shows that three of the top four variables can result in a negative PW, which would possibly change the recommended decision.

Usually, but not always, the minimum and maximum values for the PW with each variable will occur at the limits for each variable. An example exception would be a firm operating at the optimal level, which is its maximum capacity. Then doing less would result in unsatisfied customers, and attempting to do more would fail and would create unsatisfied customers. For this example exception, the best PW would be from the "no change" percentage.

If a spiderplot is built, then adding rows for the minimum and maximum values for each variable is easy (A41:I42 in Figure 2-6). If no spiderplot is built, it is usually safe to assume that the minimum and maximum PW values occur at the extreme "allowed" values for each variable. In either case the range for each variable equals the maximum PW minus the minimum.

Then the tornado diagram is a stacked bar where each variable's bar begins at its minimum value and stacks the range to equal its maximum value. The TORNADO TEMPLATE included on the CD correctly accounts for the different possible combinations of negative and positive values. Since the tornado is normally displayed from most impact at the top to least impact at the bottom, these values must be sorted. Sometimes sorting with formulas creates errors, which can be avoided by using a copy with PASTE SPECIAL/VALUES to enter values into a copy of the template.

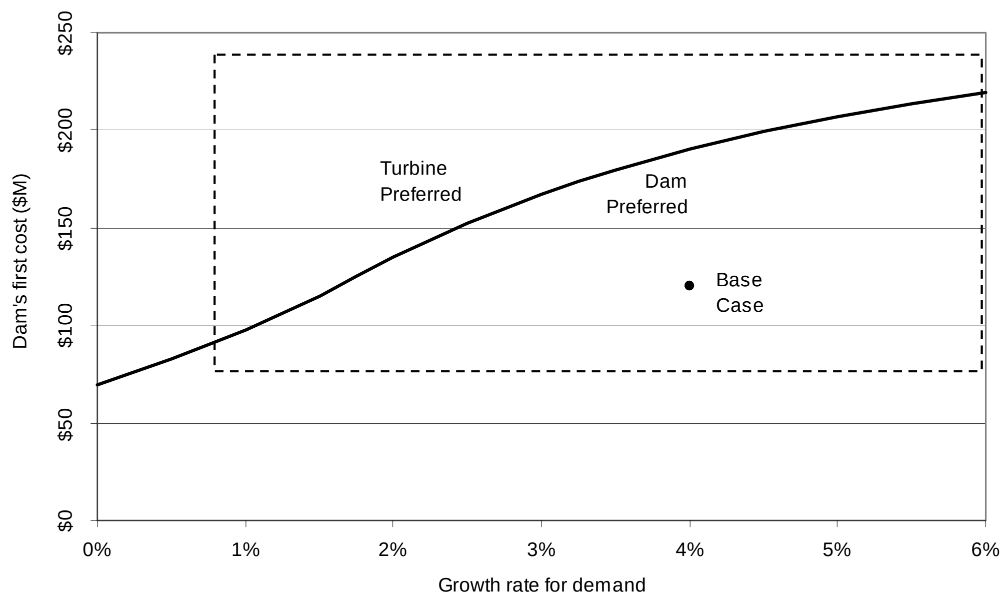
Figure 4-4 Tornado Diagram to Summarize the Relative Sensitivity of All Variables



Multiple Variables or Multiple Alternatives

Sometimes it may be important to study the simultaneous impact of two variables on the decision to be made. For example, as shown Figure 4-4, the growth rate in demand and the dam's first cost have the most negative PWs in the example case. Other choices could be based on which variable has the largest slope or unit impact (the dam's first cost), the most uncertainty (the growth rate for fuel costs, −80% to +200%), or the greatest impact on the PW (interest rate or growth rate for turbine costs). It might also be important to study together variables such as the discount rate and study horizon that are often set by policy, or somewhat arbitrary selections of the analyst (see Figure 2-3).

Figure 4-5 Two-variable Breakeven Curve

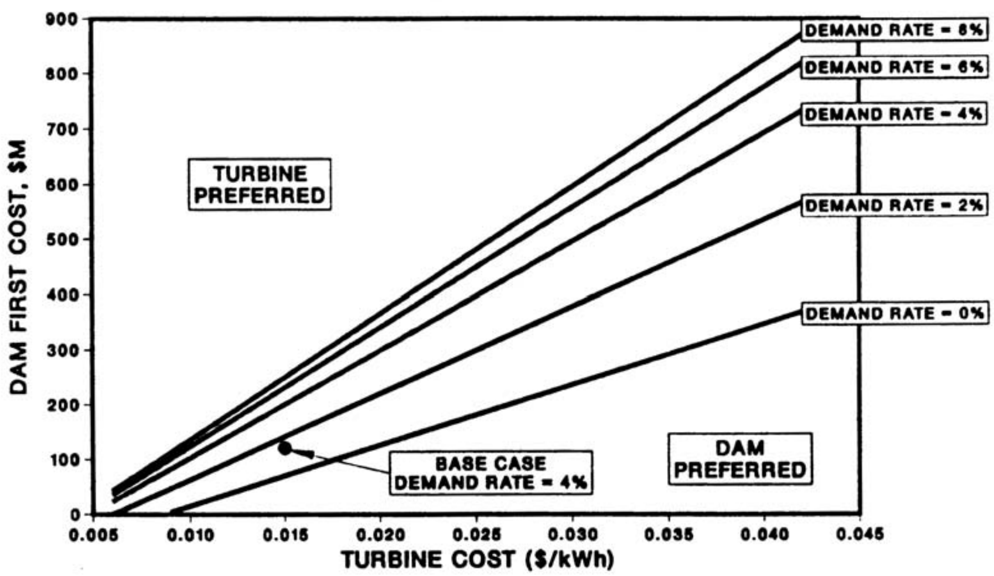


Computers can calculate tables of values that are "tiered" for any number of changing variables. However, only through limiting the number of variables to permit two-dimensional graphing is it easy to display the relationships. The breakeven or indifference line is where the PW of the proposal (go/no-go or option 1/option 2) is equal to zero. Any combination of the variables above the line will favor one option, while those below the line will favor the other option. This is shown in Figure 4-5. Note that the region within the identified lower and upper limits for each variable is shown with the dashed box.

This type of figure can also be constructed with the axes through the base case or with the axes measured as percentage change from the base. However, unlike the relative sensitivity charts, only one variable is being plotted along each axis. This allows the use of natural units, rather than percentage change for the axis variables. Since natural units are likely to fit the decision-maker's understanding better, they are a better choice when they can be used.

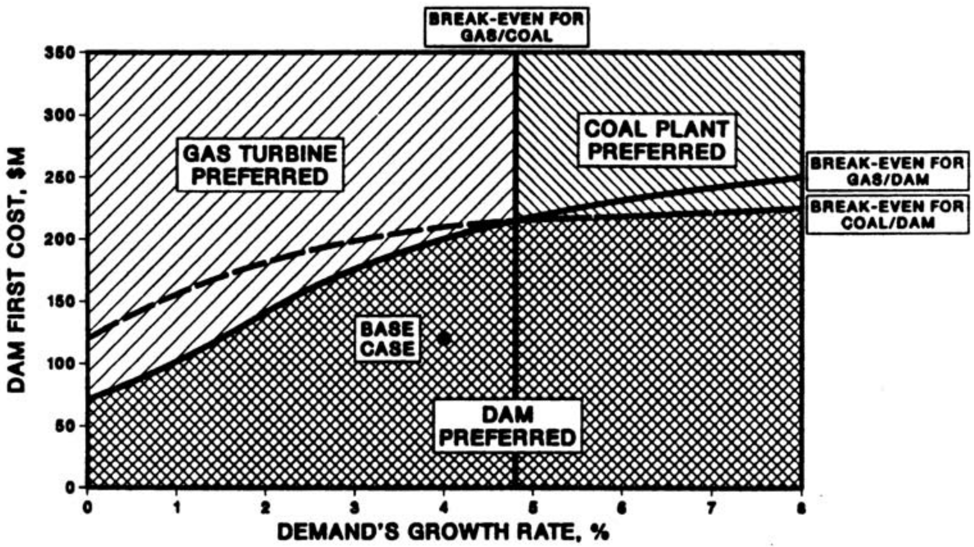
Figure 4-6 shows one way to display a sensitivity analysis for 3 variables on a 2-dimensional plot that is a series of breakeven curves. Obviously as the turbine cost increases, the dam cost can also increase before the turbine becomes the favored option. As the growth rate for demand decreases, the turbine becomes more favorable. With this graph, the analyst can determine how large a change in these three parameters can be sustained without changing the decision. If three variables are plotted, then the value of the third variable should be included in the annotation of the base case.

Figure 4-6 Multiple-variable Breakeven Curve



Similar plots with multiple alternatives, rather than multiple variables can also be constructed. Figure 4-7 is a more complex version of Chapter 2's Figure 2-4. This example adds a coal alternative with fixed and operating costs that are between the dam and turbine possibilities. Note that the breakeven line for coal and gas is vertical because this comparison is unaffected by the dam's first cost. In this type of graph each undominated alternative is identified with a portion of the feasible region.

Figure 4-7 Multiple Alternatives



Summary

When constructing graphs to display sensitivity analysis, *all* of the graphs benefit from **clear delineation of the base case and the "breakeven" values**. *Breakeven charts* are best for analyzing one variable at a time. However for comparing variables, *spiderplots* (see Figure 4-2) can display for a limited number of variables:

1. The reasonable limits of change for each variable
2. The unit impact of these changes on the PW or other measure of quality

3. The maximum impact of each variable on the PW
4. The amount of change required to cross over the breakeven line

Tornado diagrams (see Figure 4-4) can display for all variables in a model:

1. The maximum impact of each variable on the PW
2. Which of these are enough to cross over the breakeven line

In case analysis and in applying engineering economy in the real world, uncertainty in a model's values is usually inescapable. Fortunately, spreadsheets give us the tools to efficiently and effectively describe the impact of this uncertainty.

For Further Reading

Eschenbach, T. G. and L. S. McKeague, "Exposition on Using Graphs for Sensitivity Analysis," *The Engineering Economist*, Vol. 37 No. 4, Summer 1989, pp. 315–333

Eschenbach, Ted, "Spiderplots vs. Tornado Diagrams for Sensitivity Analysis," *Interfaces*, Vol. 22 #6, Nov.-Dec. 1992, The Institute for Management Sciences, pp. 40–46

Eschenbach, Ted, *Engineering Economy: Applying Theory to Practice* 2nd, Oxford University Press, 2003, chapter 17

Tufte, Edward R., *Beautiful Evidence*, Graphics Press, 2006