THE CHEMIST'S TOOLKIT 27 Dipolar magnetic fields

Standard electromagnetic theory gives the magnetic field at a point $\boldsymbol{r}$ from a point magnetic dipole $\boldsymbol{\mu}$ as

$$\mathcal{B} = -\frac{\mu_0}{4\pi r^3}\left(\boldsymbol{\mu} - \frac{3(\boldsymbol{\mu}\cdot\boldsymbol{r})\boldsymbol{r}}{r^2}\right) \tag{27.1}$$

where μ_0 is the vacuum permeability (a fundamental constant with the defined value $4\pi \times 10^{-7}\,\mathrm{T^2\,J^{-1}\,m^3}$). The component of magnetic field in the z-direction is

$$\mathcal{B}_z = -\frac{\mu_0}{4\pi r^3}\left(\mu_z - \frac{3(\boldsymbol{\mu}\cdot\boldsymbol{r})z}{r^2}\right) \tag{27.2}$$

with $z = r\cos\theta$, the z-component of the distance vector $\boldsymbol{r}$. If the magnetic dipole is also parallel to the z-direction, it follows that

$$\mathcal{B}_z = -\frac{\mu_0}{4\pi r^3}\left(\overbrace{\mu}^{\mu_z} - \frac{3\overbrace{(\mu r\cos\theta)}^{\boldsymbol{\mu}\cdot\boldsymbol{r}}\overbrace{(r\cos\theta)}^{z}}{r^2}\right) = -\frac{\mu\mu_0}{4\pi r^3}(1-3\cos^2\theta) \tag{27.3}$$