

THE CHEMIST'S TOOLKIT 23 Determinants

A 2×2 determinant is the entity

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \quad \text{2x2 Determinant (23.1a)}$$

A 3×3 determinant is evaluated by expanding it as a sum of 2×2 determinants:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ = a(ei - fh) - b(di - fg) + c(dh - eg) \quad \text{3x3 Determinant (23.1a)}$$

Note the sign change in alternate columns (b occurs with a negative sign in the expansion). An important property of a determinant is that if any two rows or any two columns are interchanged, then the determinant changes sign:

Exchange columns:

$$\begin{vmatrix} b & a \\ d & c \end{vmatrix} = bc - ad = -(ad - bc) = - \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Exchange rows:

$$\begin{vmatrix} c & d \\ a & b \end{vmatrix} = cb - da = -(ad - bc) = - \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

An implication is that if any two columns or rows are identical, then the determinant is zero.